Outline

- Utility and Demand
- Axioms of Consumer Choice
- Utility Function
- Indifference Curve
- Examples of Utility Function and Indifference Curves
- Utility Maximization in Energy Modeling

Utility and Demand

Utility

Utility is a measure of the value which consumers of a product or service place on that product or service

Demand

Demand is a reflection of this measure of value, and is represented by price per quantity of output

Utility and Demand

The Income Effect

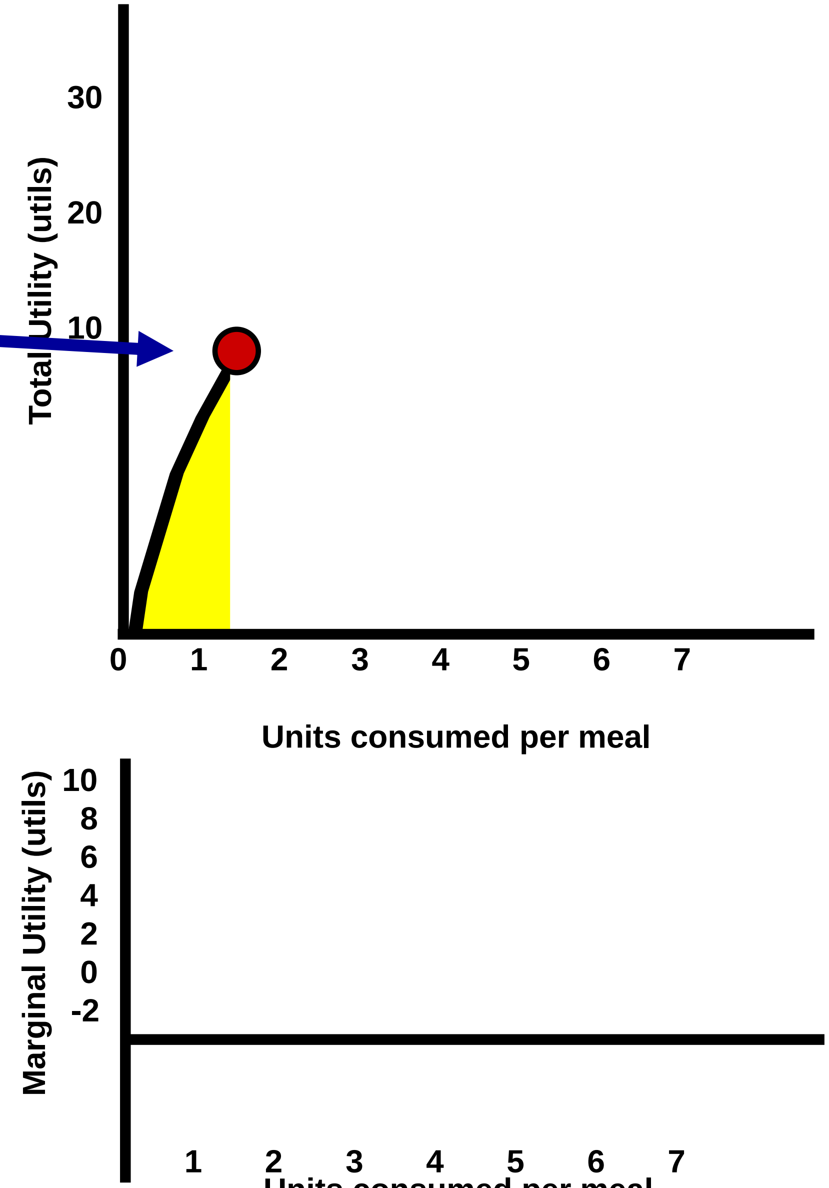
A lower price frees income for additional purchases - and vice versa

The Substitution Effect

A lower price relative to other goods attracts new buyers - and vice versa

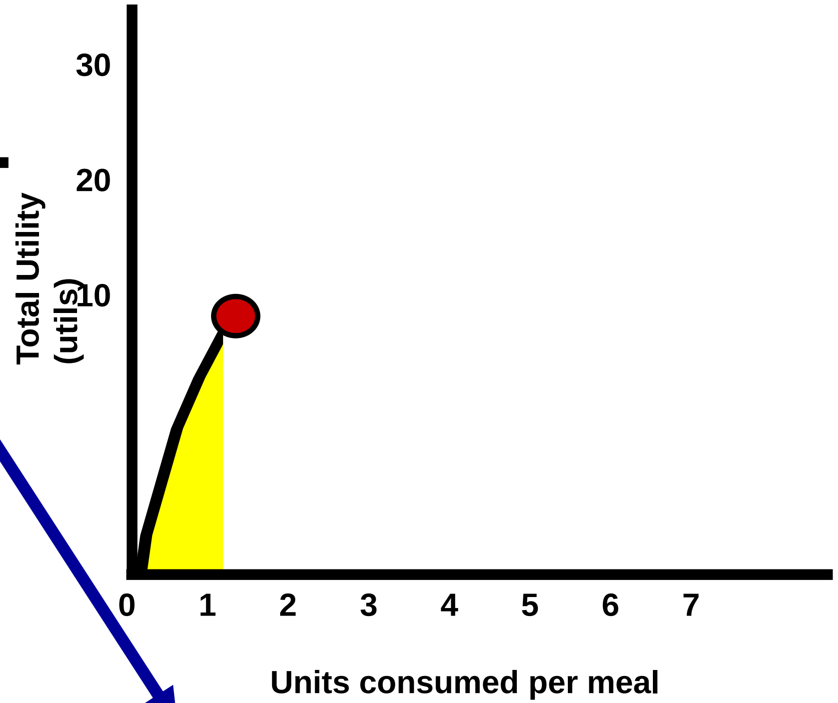
Utility and Demand

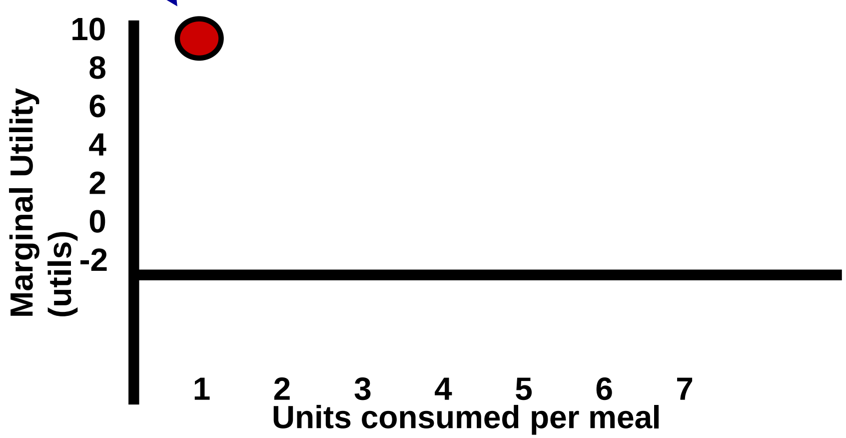
Tacos consumed per meal	Total Utility, Utils
0	0
1	10



Utility and Demand

Tacos consumed per meal	Total Utility, Utils	Marginal Utility, Utils
0	0	
1	10	10



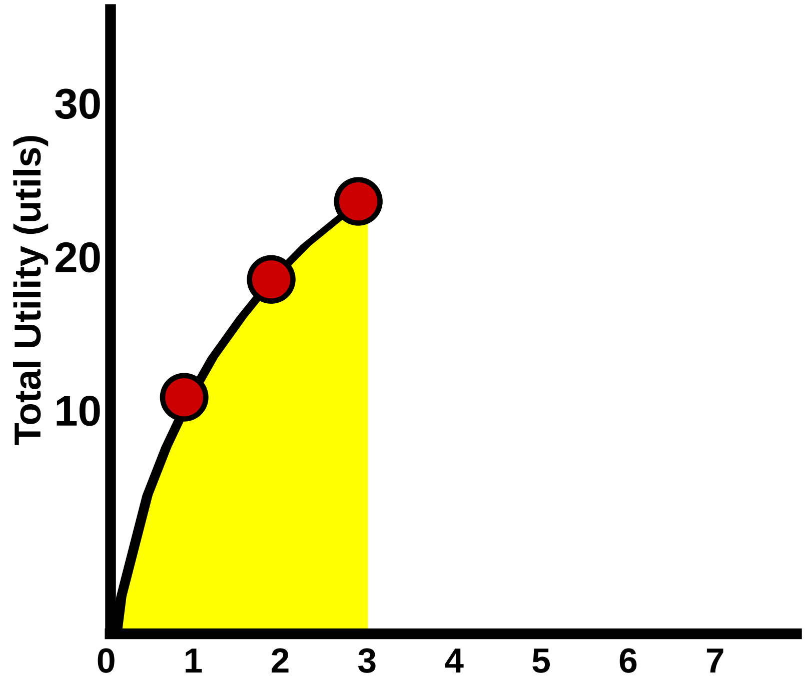


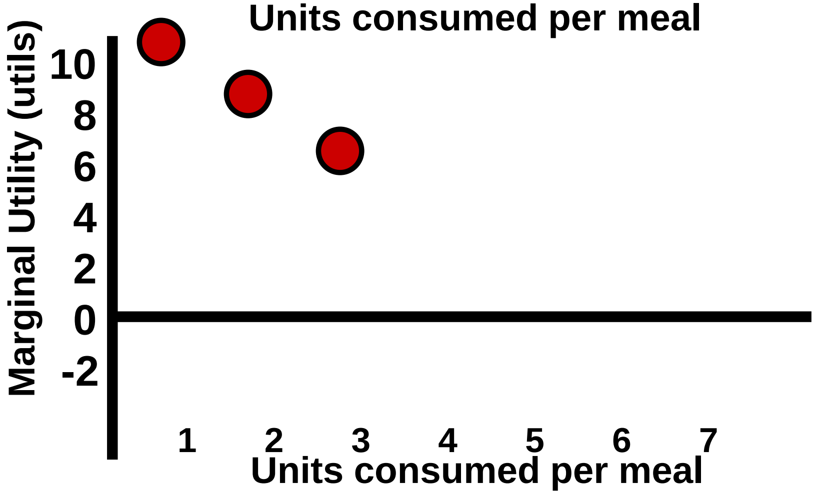
Utility and Demand

Tacos consumed per meal	Total Utility, Utils	Marginal Utility, Utils
0	0	
1	10	10
2	18	8

Utility and Demand

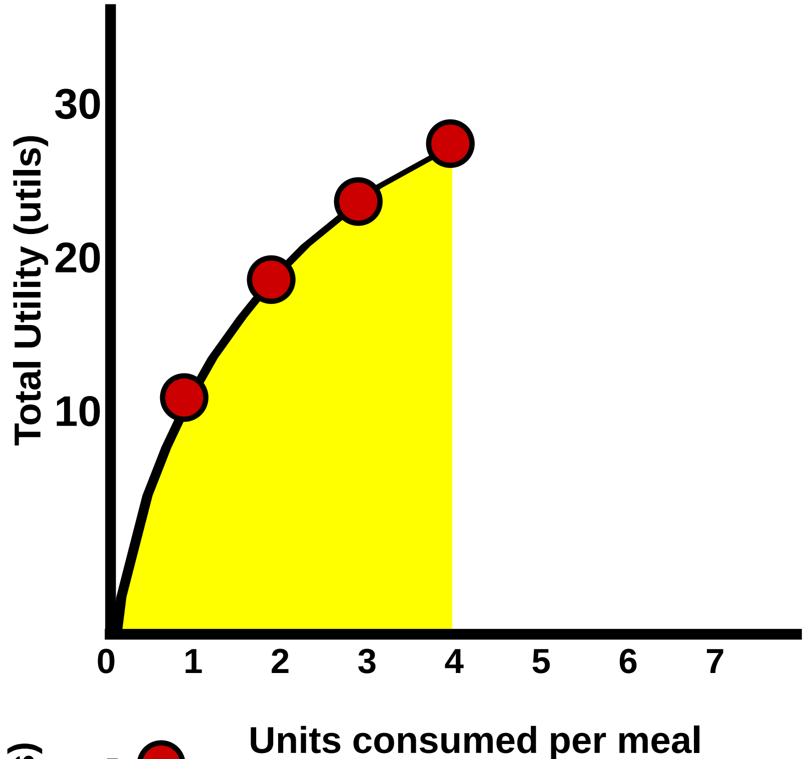
Tacos consumed per meal	Total Utility, Utils	Marginal Utility, Utils
0	0	
1	10	10
2	18	8
3	24	6

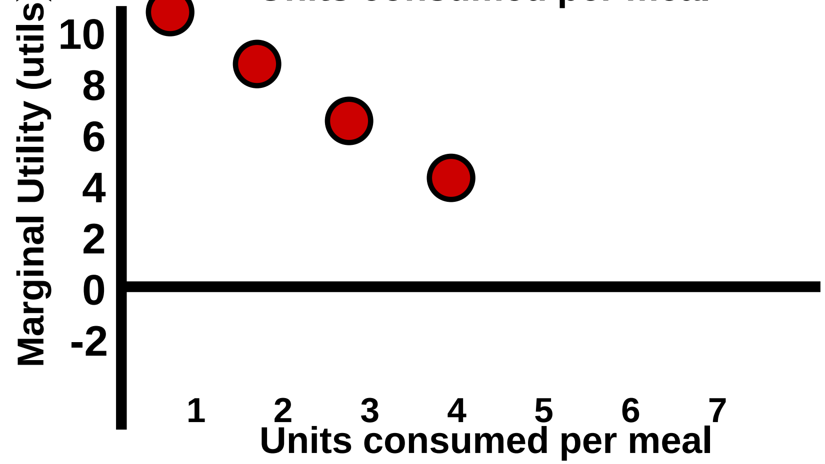




Utility and Demand

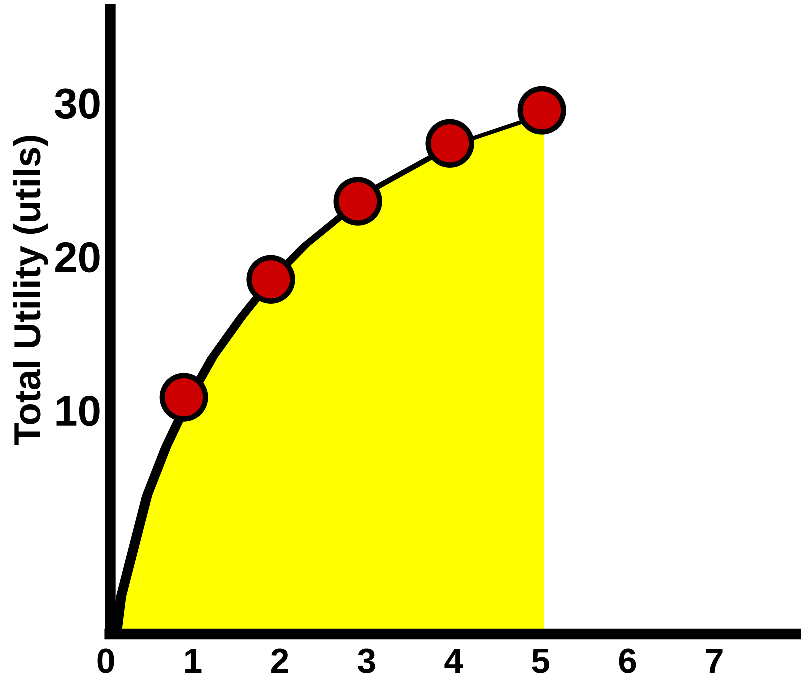
Tacos consumed per meal	Total Utility, Utils	Marginal Utility, Utils
0	0	
1	10	10
2	18	8
3	24	6
4	28	4

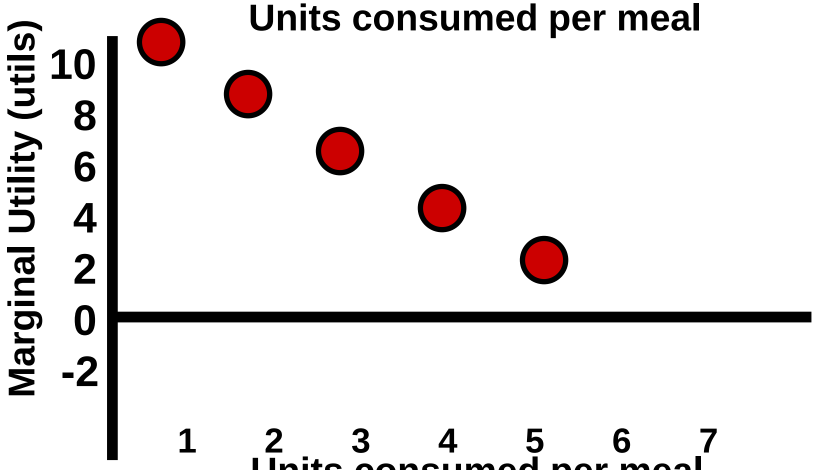




Utility and Demand

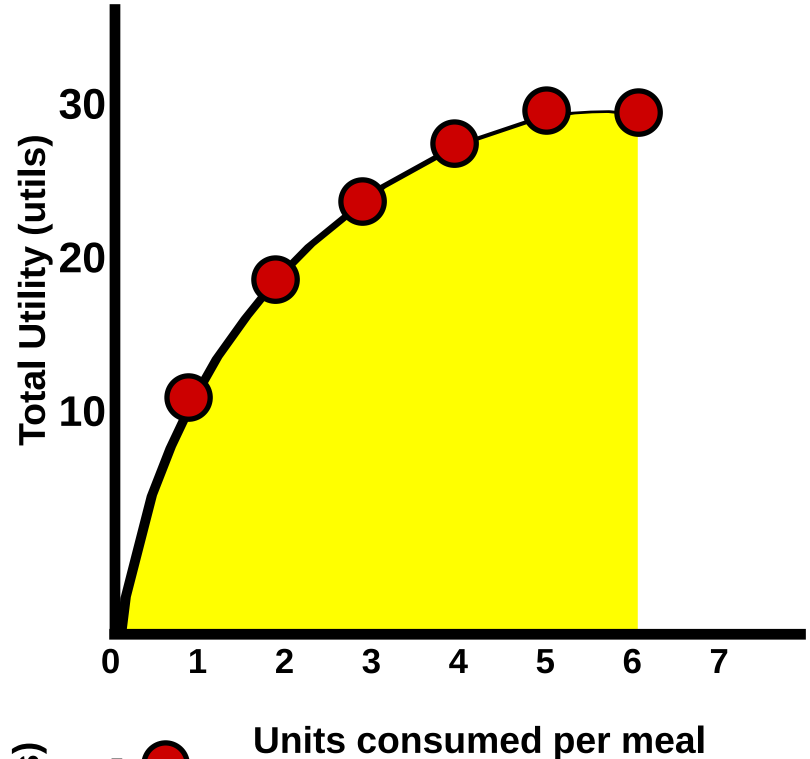
Tacos consumed per meal	Total Utility, Utils	Marginal Utility, Utils
0	0	
		10
1	10	
		8
2	18	
		6
3	24	
		4
4	28	
		2
5	30	

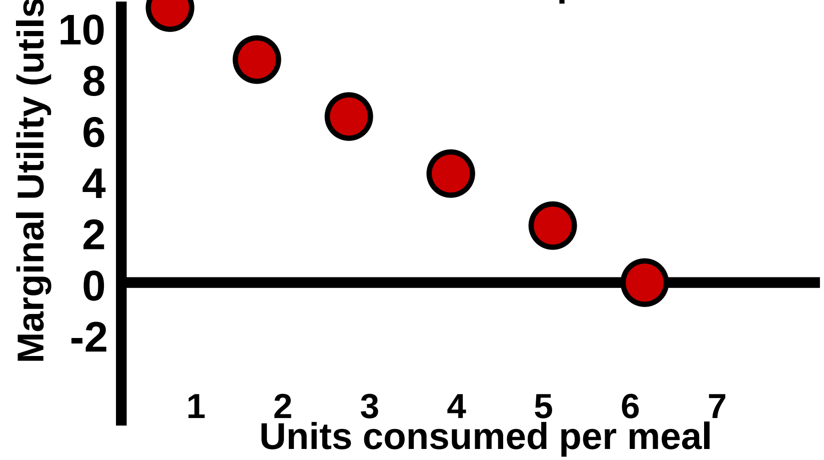




Utility and Demand

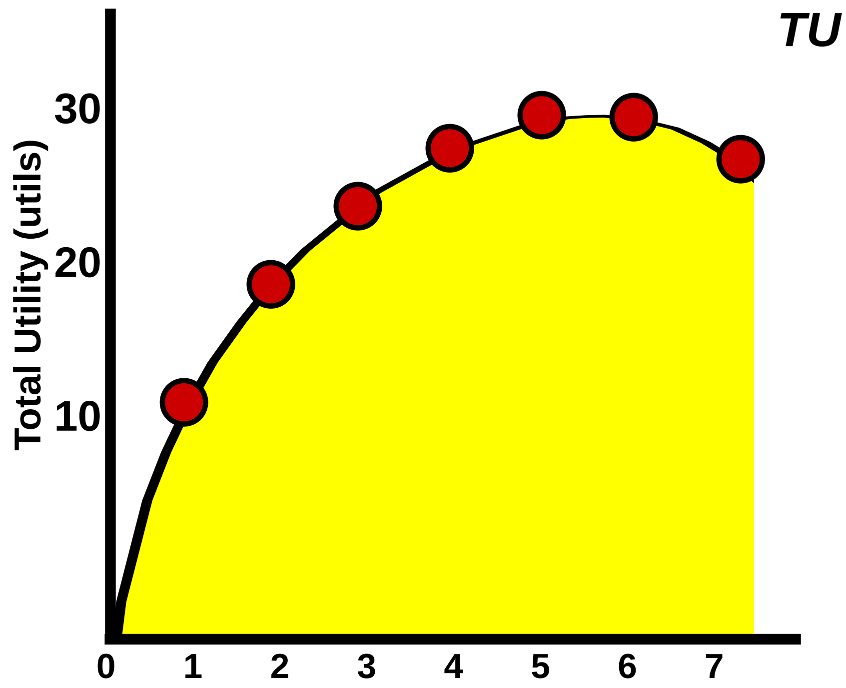
Tacos consumed per meal	Total Utility, Utils	Marginal Utility, Utils
0	0	
1	10	10
2	18	8
3	24	6
4	28	4
5	30	2
6	30	0

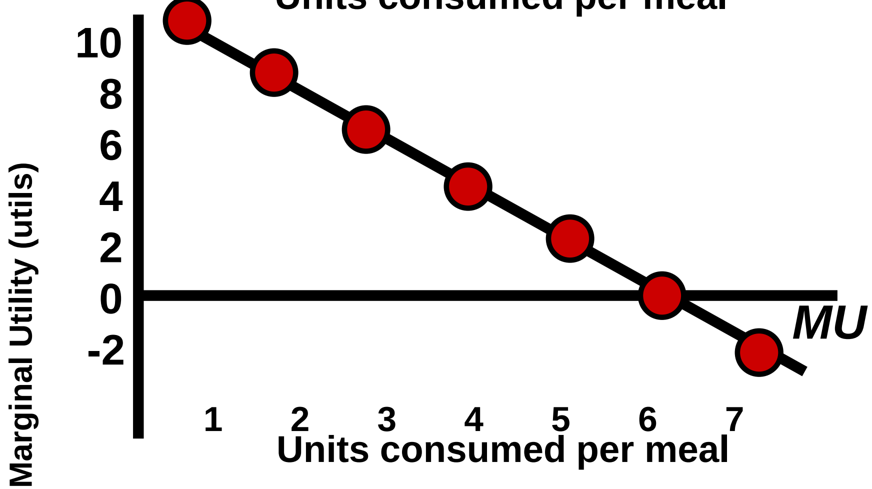




Utility and Demand

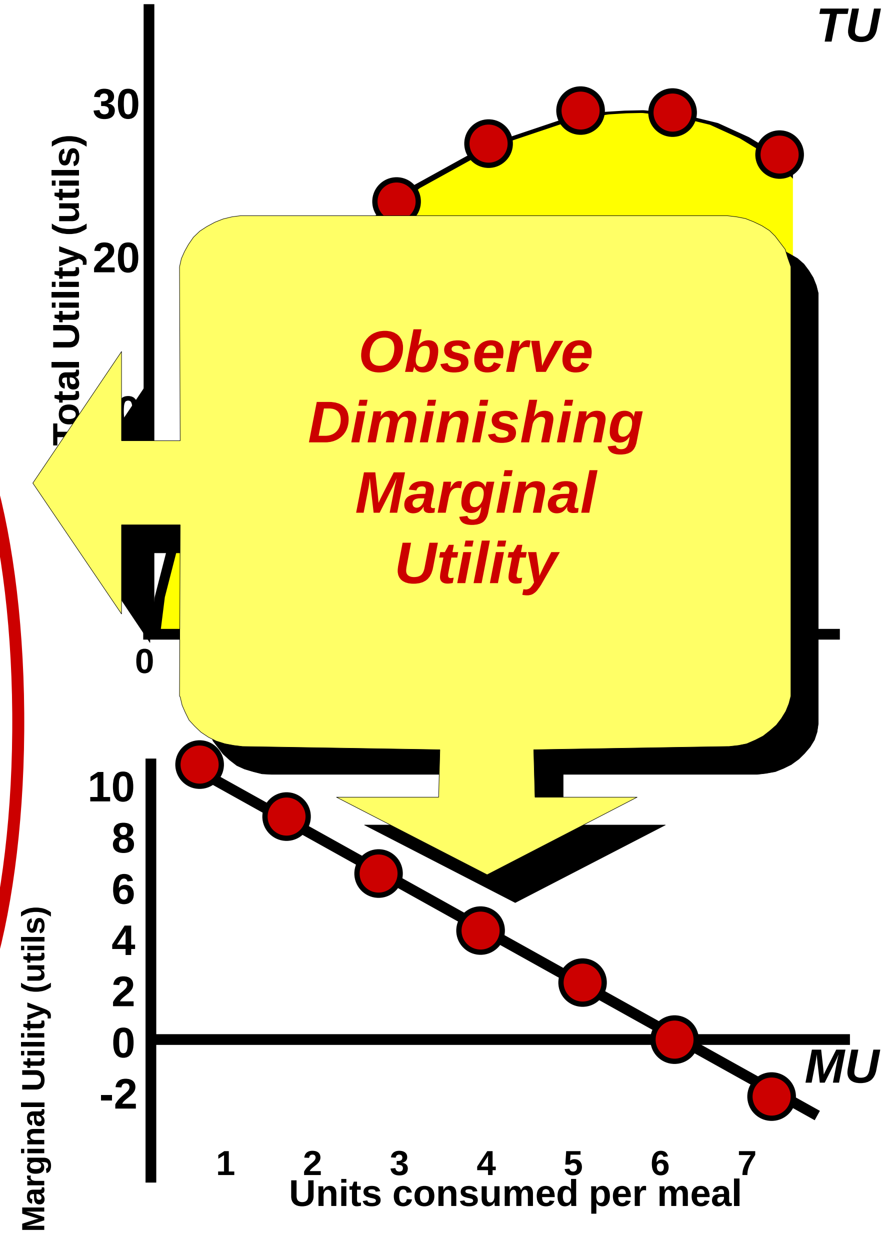
Tacos consumed per meal	Total Utility, Utils	Marginal Utility, Utils
0	0	
1	10	10
2	18	8
3	24	6
4	28	4
5	30	2
6	30	0
7	28	-2





Utility and Demand

Tacos consumed per meal	Total Utility, Utils	Marginal Utility, Utils
0	0	
1	10	10
2	18	8
3	24	6
4	28	4
5	30	2
6	30	0
7	28	-2



Utility and Demand

Utility Maximizing Rule

The consumer's income should be allocated so that the last unit spent on each product yields the same amount of marginal utility.

Utility and Demand

$ 10 income

	Product A: Price = $1		**Product B: Price = $2**	
Unit of product	***Marginal utility, utils***	***Marginal utility per dollar (MU/price)***	***Marginal utility, utils***	***Marginal utility per dollar (MU/price)***
First	**10**	**10**	**24**	**12**

How should the $10 income be allocated?

Utility and Demand

$ 10 income

	Product A: Price = $1		Product B: Price = $2	
Unit of product	*Marginal utility, utils*	*Marginal utility per dollar (MU/price)*	*Marginal utility, utils*	*Marginal utility per dollar (MU/price)*
First	10	10	24	12

Examine the two marginal utilities

Utility and Demand

$ 10 income

	Product A: Price = $1		**Product B: Price = $2**	
Unit of product	*Marginal utility, utils*	*Marginal utility per dollar (MU/price)*	*Marginal utility, utils*	*Marginal utility per dollar (MU/price)*
First	**10**	**10**	**24**	**12**

Examine the two marginal utilities

…per dollar

Utility and Demand

$ 10 income

	Product A: Price = $1		**Product B: Price = $2**	
Unit of product	***Marginal utility, utils***	***Marginal utility per dollar (MU/price)***	***Marginal utility, utils***	***Marginal utility per dollar (MU/price)***
First	**10**	**10**	**24**	**12**

Decision: Buy 1 Product B for $2

Utility and Demand

$ 10 income

Unit of product	Product A: Price = $1 *Marginal utility, utils*	*Marginal utility per dollar (MU/price)*	Product B: Price = $2 *Marginal utility, utils*	*Marginal utility per dollar (MU/price)*
First	10	10	24	12
Second	8		20	10
Third				9
Fo				
Si				
Seven				2

What next?

Utility and Demand

$ 10 income

Unit of product	*Marginal utility, utils*	*Marginal utility per dollar (MU/price)*	*Marginal utility, utils*	*Marginal utility per dollar (MU/price)*
	Product A: Price = $1		**Product B: Price = $2**	
First	10	10 ✓	24	12 ✓
Second	8		20	10 ✓
Third				9
Fo				
Si				
Seven				2

What next?

Buy one of each

Utility and Demand

$ 10 income

Unit of product	*Product A: Price = $1* Marginal utility, utils	*Product A: Price = $1* Marginal utility per dollar (MU/price)	*Product B: Price = $2* Marginal utility, utils	*Product B: Price = $2* Marginal utility per dollar (MU/price)
First	10	10	24	12
Second	8	8	20	10
Third	7	7	18	9
Fourt				8
Fi				6
Si				3
Seven				2

and then... ($5 left)

Utility and Demand

$ 10 income

Unit of product	**Product A: Price = $1** *Marginal utility, utils*	*Marginal utility per dollar (MU/price)*	**Product B: Price = $2** *Marginal utility, utils*	*Marginal utility per dollar (MU/price)*
First	10	10	24	12
Second	8	8	20	10
Third	7	7	18	9
Fourt	[illegible]	[illegible]	[illegible]	8
Fi	[illegible]	[illegible]	[illegible]	6
Si	[illegible]	[illegible]	[illegible]	3
Seven	[illegible]	[illegible]	[illegible]	2

third unit of product B

Utility and Demand

$ 10 income

Unit of product	*Product A: Price = $1* Marginal utility, utils	*Product A: Price = $1* Marginal utility per dollar (MU/price)	*Product B: Price = $2* Marginal utility, utils	*Product B: Price = $2* Marginal utility per dollar (MU/price)
First	10	10	24	12
Second	8	8	20	10
Third	7	7	18	9
Fourth	[illegible]	[illegible]	16	8
[illegible]	[illegible]	[illegible]	[illegible]	6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	2

$3 left...

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Utility and Demand

$ 10 income

Unit of product	Product A: Price = $1 Marginal utility, utils	Product A: Price = $1 Marginal utility per dollar (MU/price)	Product B: Price = $2 Marginal utility, utils	Product B: Price = $2 Marginal utility per dollar (MU/price)
First	10	10	24	12
Second	8	8	20	10
Third	7	7	18	9
Fourth	[illegible]	[illegible]	16	8
[illegible]	[illegible]	[illegible]	[illegible]	6
[illegible]	[illegible]	[illegible]	[illegible]	3
[illegible]	[illegible]	[illegible]	[illegible]	2

$3 left...

Buy both!

Utility and Demand

$ 10 income

Unit of product	*Marginal utility, utils* (Product A: Price = $1)	*Marginal utility per dollar (MU/price)* (Product A: Price = $1)	*Marginal utility, utils* (Product B: Price = $2)	*Marginal utility per dollar (MU/price)* (Product B: Price = $2)
First	10	10	24	12
Second	8	8	20	10
Third	7	7	18	9
Fourt			6	8
Fi				6
Six				3
Seven			4	2

Income is gone...
the last dollar spent on each good gave the same utility (8) per dollar

Utility and Demand

Algebraic Restatement of the Utility Maximization Rule

$$\frac{\text{MU of product A}}{\text{Price of A}} = \frac{\text{MU of product B}}{\text{Price of B}}$$

$$\frac{\text{8 Utils}}{\$1} = \frac{\text{16 Utils}}{\$2}$$

Utility and Demand

Deriving the Demand Schedule and Curve

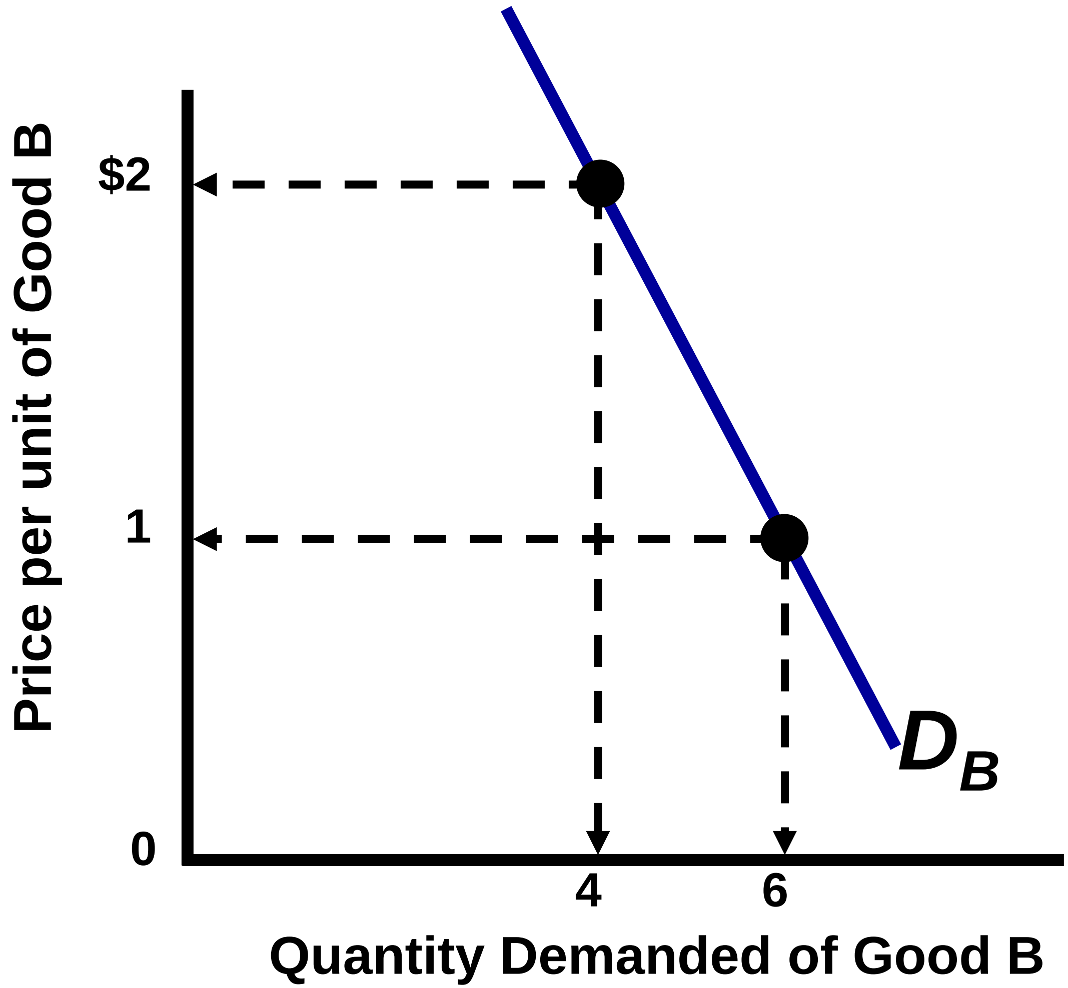
Create a demand schedule from the purchase decisions as the price of the product is varied ...

Price per unit of B	*Quantity Demanded*
$2	**4**
1	**6**

Graphically…

Utility and Demand

Deriving the Demand Schedule and Curve



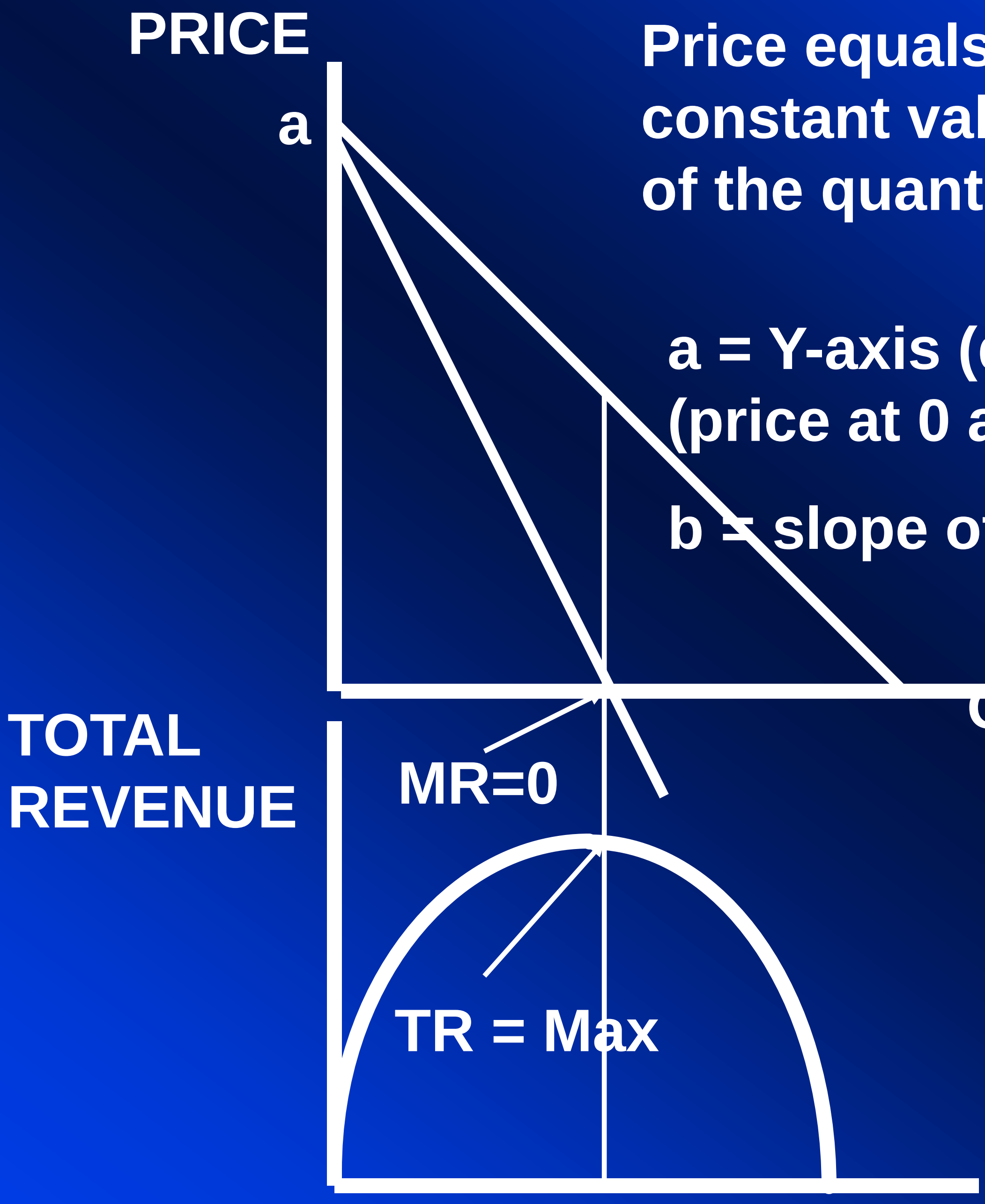
Price equals some
constant value minus some multiple
of the quantity demanded:

$$p = a - b\,D$$

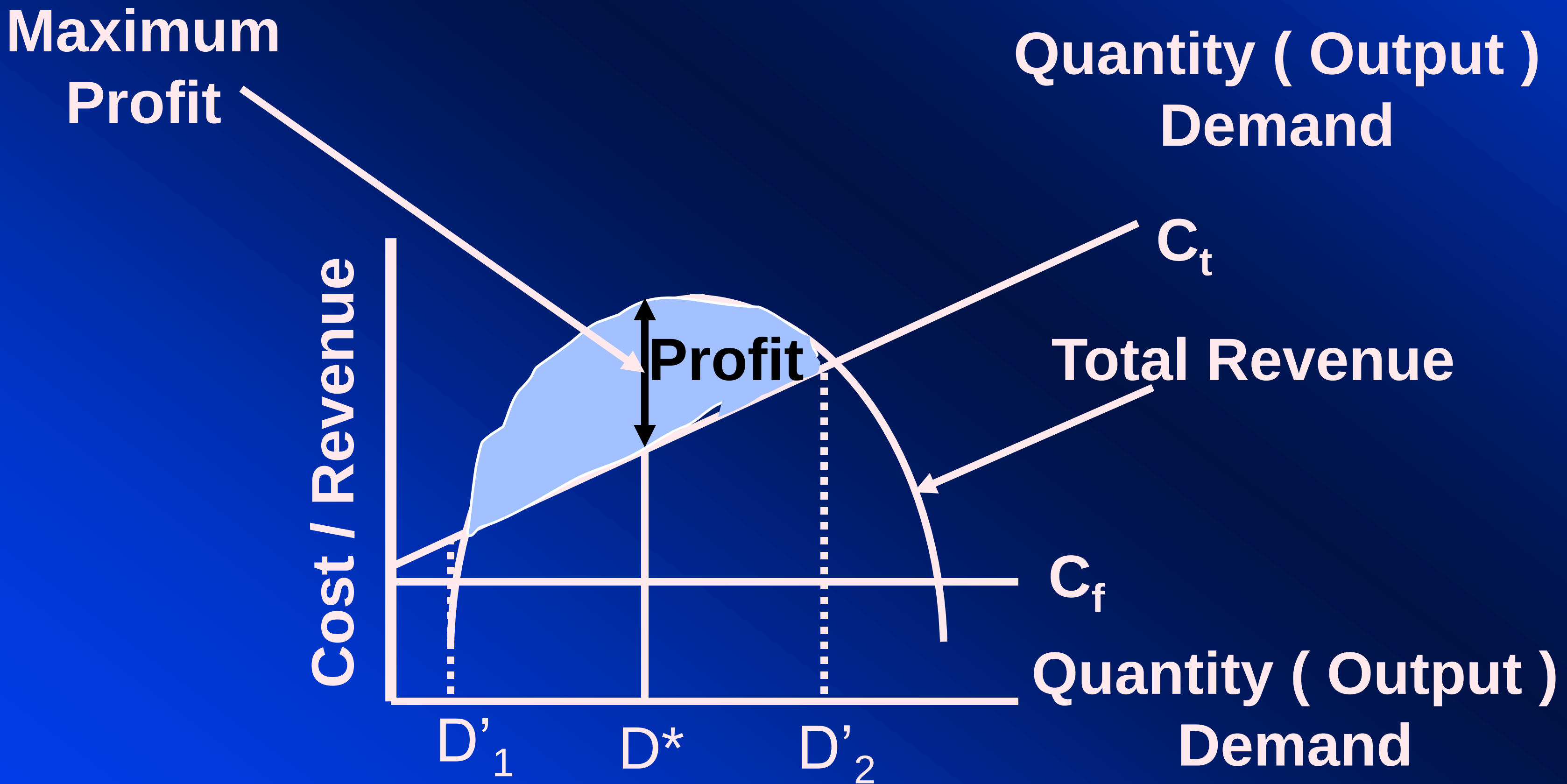
a = Y-axis (quantity) intercept,
(price at 0 amount demanded);

b = slope of the demand function;

$$D = (a - p) / b$$



Profit is maximum where Total Revenue exceeds Total Cost by greatest amount



D'1 and D'2 are breakeven points

PROFIT MAXIMIZATION
D*

- Occurs where total revenue exceeds total cost by the greatest amount;
- Occurs where marginal cost = marginal revenue;
- Occurs where $d\text{TR}/d\text{D} = d\text{C}_t /d\text{D}$;
- $\text{D}^* = [\ a - c_v\ ] / 2b$

BREAKEVEN POINT
D'_1 and D'_2

- Occurs where $TR = C_t$
- $aD - bD^2 = C_f + (c_v) D$
- $- bD^2 + [a - c_v] D - C_f = 0$
- Using the quadratic formula: $D' =$

$$\frac{-[a - c_v] \pm \{[a - c_v]^2 - 4(-b)(-C_f)\}^{1/2}}{2(-b)}$$

Utility and Demand

Consumer Choice

- **Rational Behavior**
- **Clear-Cut Preferences**
- **Responds to Price Changes**
- **Subject to a Budget Constraint**

Utility Function: Theoretical Underpinnings

Axioms of Consumer Choice

1. Completeness

Consumer choice theory is based on the assumption that the consumer fully understands his/her own preferences, allowing for a simple but accurate comparison between any two bundles of goods presented.

A person can compare any two bundles of goods *A and B in such a way that it leads to one of the three following results: he or she (i) prefers A over B, or (ii) prefers B over A, or (iii) both A and B are the same (they are indifferent)*

Three cases:

i. A is preferred to B: AðB
ii. B is preferred to A: B>A
iii. A and B are the same: A~B

This axiom of completeness **rules out the possibility that the consumer cannot compare** between two different baskets of commodities.

Axioms of Consumer Choice

2. Transitivity

Consider any three bundles of goods *A, B, and C.* ***If a consumer prefers A to B, and also prefers B to C, he or she must prefer A to C.*** Similarly, a person who is indifferent between A and B, and is also indifferent between B and C, must be indifferent between A and C.

This is the ***consistency*** assumption. This assumption **eliminates the possibility of intersecting indifference curves.**

Axioms of Consumer Choice

The axiom of **completeness** and the axiom of **transitivity** are the two **most basic assumptions** towards people's preferences. They are derived l*ogically* because, without either one of them, an economic analysis cannot be performed.

If someone's behavior satisfies the axiom of completeness and the axiom of transitivity, we know that he/she can rank any bundles of goods.

Proposition: A consumer can consistently rank all bundles of goods in order of preference.

Axioms of Consumer Choice

Further assumptions, however, are still needed to define a typical indifference curve based on a "well-behaved" preference. These are defined in the following. They are based on ***empirical observations*** towards people's choices and preferences.

Axioms of Consumer Choice

3. Non-satiation

This is the "more is always better" assumption

If a consumer is offered two almost identical bundles A and B, but where B includes more of one particular good, the consumer will choose B

You may think of a counter-example that after you have eaten 10 ice-creams, you will not want even a single one. But don't forget that in a market you can always trade those additional ice-creams for money and then purchase other goods. Thus "the more, the better" generally holds.

Non-satiation is **not a necessary but a convenient assumption**. It precludes circular indifference curves and avoids unnecessary complications in mathematical models.

Axioms of Consumer Choice

4. People's Preference of Variety

Assume any weight t between 0 and 1, if there are two bundles of commodities (x_1,y_1) and (x_2,y_2) such that $(x_1,y_1) \sim (x_2,y_2)$, we have

$$(tx_1 + (1-t)x_2\ ,\ ty_1 + (1-t)\ y_2) \geq (x_1,y_1)$$

This means, given any two bundles that are indifferent to a consumer, the **mixture of these two bundles is always as good as any of them.**

Utility Function

A utility function is a mathematical expression that shows the relationship between utility values of every possible bundle of goods.

Suppose there are three commodity baskets A, B and C, and the preference ordering towards these three baskets is: A>B>C

When we assign a utility function to this preference ordering, it can simply be considered as a transformation from the above expression:
$U(A)>U(B)>U(C)$

Utility Function

Cardinal vs Ordinal

Cardinal Utility Function:

According to this approach U(A) is a cardinal number, that is:

U : consumption bundle →R1 measured in ”utils”

Assigning numerical values to the amount of satisfaction

Ordinal Utility Function:

More general than cardinal utility function

U provides a ”ranking” or ”preference ordering” over bundles.

Not assigning numerical values to the amount of satisfaction but indicating the order of preferences

Utility Function

Cardinal vs Ordinal

The problem with cardinal utility functions comes from the difficulty in finding the appropriate measurement index (metric).

Is 1 util for person 1 equivalent to 1 util for person 2 ?
What is the proper metric for comparing U1 vs U2 ?
How can interpersonal comparisons be made ?

By being unit-free ordinal utility functions avoid these problems.

All that matters about utility as far as choice behavior is concerned is whether one bundle has a higher utility than another – how much higher doesn't really matter.

Therefore, **Ordinal Utility Functions** are used in demand/consumer theory

Utility Function

Since only the **ranking** of the bundles matters, there can be no unique way to assign utilities to bundles of goods.

If we can find a way to assign utility numbers to bundles of goods, we can find an infinite number of ways to do it – simply by multiplying the utility measure by any positive number.

Any monotonic transformation of $U(x_1,y_1)$ is just as good a way to assign utilities as $U(x_1,y_1)$ itself.

Geometrically, a utility function is a way to label **indifference curves** in such a way that higher indifference curves get assigned larger numbers.

Indifference Curve

From Preference to Indifference Curve

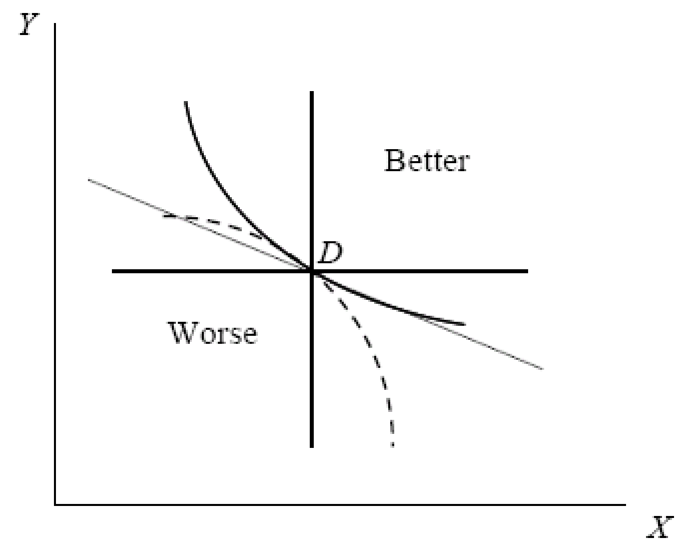
Definition of Indifference Curve: An **indifference curve** is a graph showing different bundles of goods between which a consumer is *indifferent.* That is, at each point on the curve, the consumer has no preference for one bundle over another. **Every single indifference curve represents a certain level of utility.** One can equivalently refer to each point on the indifference curve as rendering the same level of utility (satisfaction) for the consumer.

Indifference Curve

1. Characteristics: Negative sloping

As quantity consumed of one good (X) increases, total satisfaction would increase if not offset by a decrease in the quantity consumed of the other good (Y).

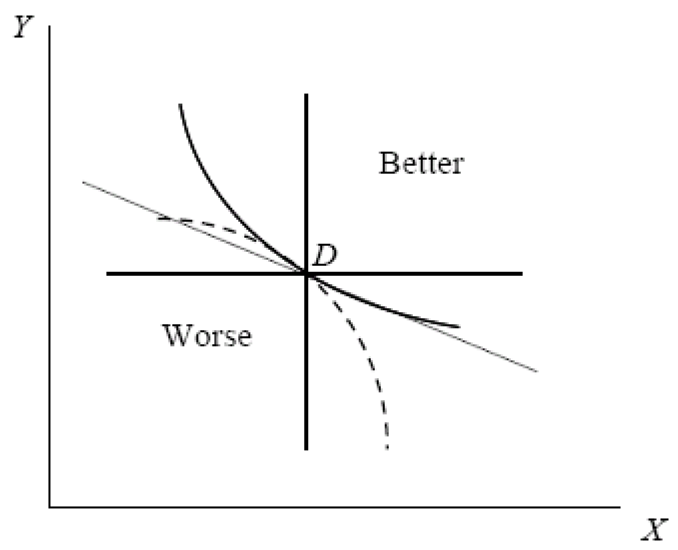
If we randomly spot a certain point, D, on a diagram representing two goods, we will see that any combinations of these two goods that lie in the northeast of this point are better; symmetrically, any combinations that lie in the southwest of this point are less desired.



Indifference Curve

1. Characteristics: Negative sloping

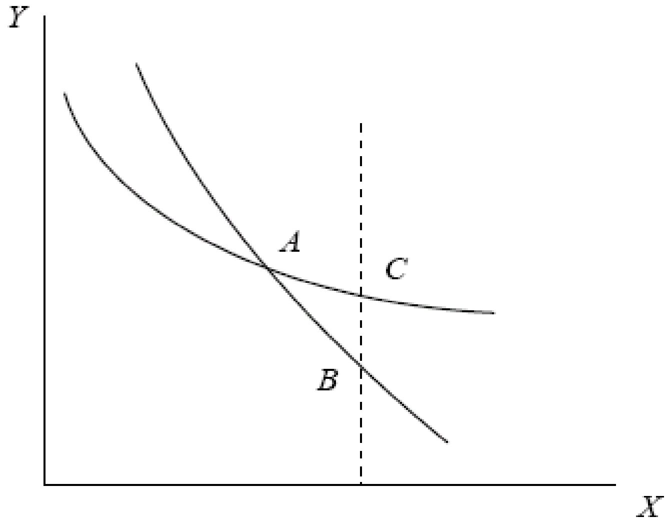
If we connect all dots representing the same level of utility as point D and form a curve, the curve should go from northwest to southeast. The curve that connects these points, by definition, is an indifference curve.



Indifference Curve

2. Characteristics: No intersection, parallelity

All points on an indifference curve are ranked equally preferred and ranked either more or less preferred than every other point not on the curve. If this characteristic doesn't hold, it violates the axiom of transitivity. Suppose two indifference curves intersect each other at point A. We can find two points, one on each indifference curve in the way that they have the same amount of good X but C has more of good Y than B.



Since A and B are on the same indifference curve, we have A~B, and at the same time, we also have A~C. Applying the axiom of transitivity, we immediately see that B~Cshould hold. But from the axiom of non-satiation, we have C>B. A conflict appears. **As a result, indifference curves can never intersect with each other.**

Indifference Curve

3. Characteristics: Completeness

There is an infinite number of indifference curves which cover the whole area of the axiom panel . This relates to the axiom of completeness. Since an individual can rank any bundle of goods, and every point on the axiom panel represents one certain basket of goods, then apparently every point should have one and only one indifference curve go through it.

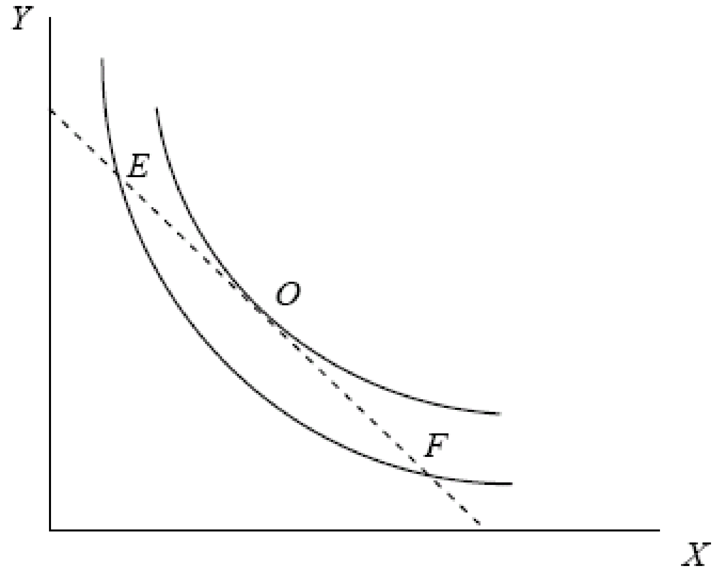
Indifference Curve

4. Characteristics: Convexity

This is the result of the fact that **people prefer variety**. On the graph below, point E represents basket (x_1,y_1) and point F represents basket (x_2,y_2). Any point on the dotted line between E and F can be expressed by

$$(tx_1 + (1\text{-}t)x_2\ ,\ ty_1 + (1\text{-}t)\ y_2) \geq (x_1,y_1)$$

by applying different value of t. Because people prefer variety, any point on the dotted line that between E and F has a higher utility than either E or F, and thus points that have the same utility as E or F (which means they are on the same indifference curve as E and F) lie on the southwest of the dotted line.



Indifference Curve

5. Characteristics: Diminishing Marginal Utility

As more of a good is consumed total utility increases at a decreasing rate - additions to utility per unit consumption are successively smaller. Thus as you move down the indifference curve you are trading consumption of units of Y for additional units of X.

The absolute value of the slope of the indifference curve represents the **Marginal Rate of Substitution.**

Indifference Curve

5. Characteristics: Diminishing Marginal Utility

Definition of Marginal Rate of Substitution (MRS): The measurement of how many units of good Y a consumer would be willing to give up to get one additional unit of good X while the consumer keeps his or her level of satisfaction (utility).

Mathematically, we can calculate MRS with the help of Marginal Utility

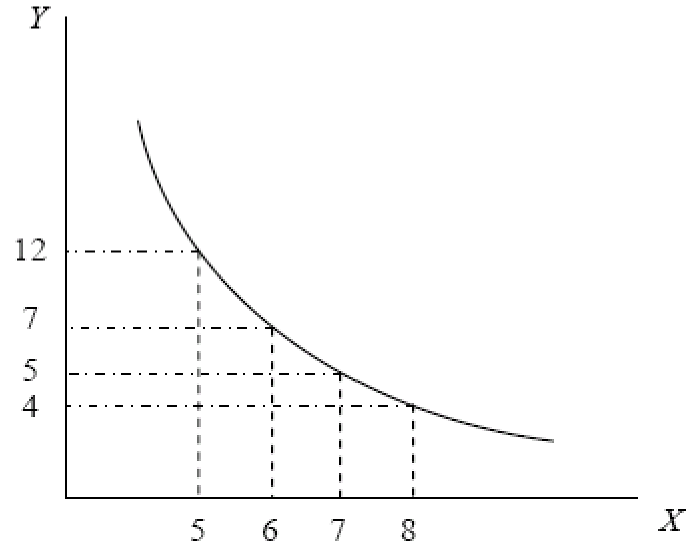
$$MRS_{XY} = \frac{MU_X}{MU_Y}$$

Indifference Curve

5. Characteristics: Diminishing Marginal Utility

An important property of MRS is called "**Diminishing Marginal Rate of Substitution**".

This captures the fact that while you are getting more and more of a certain good, say X, you are less likely to give up the other good, Y. For example, if you are very hungry, you may be willing to pay 12 $ for a hamburger (Y). But when you are almost full, you don't even want to pay 4 $ for the same hamburger.

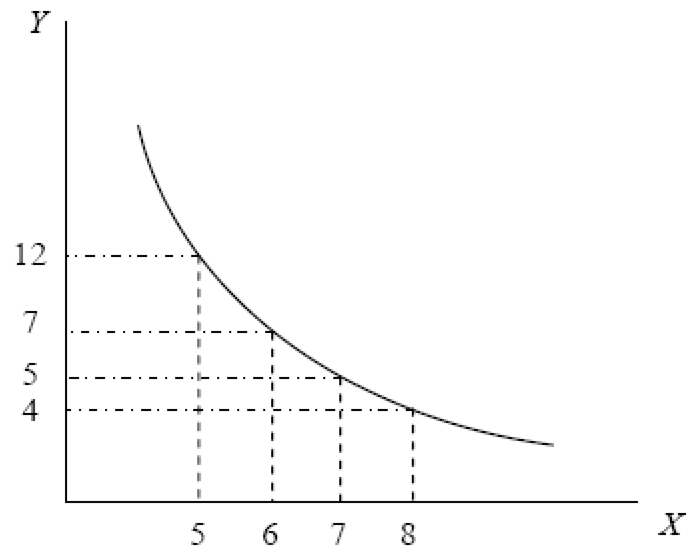


This graph also shows another way to calculate the MRS: $MRS_{XY} = \frac{\Delta Y}{\Delta X}$

Indifference Curve

5. Characteristics: Diminishing Marginal Utility

In this graph, ΔX always equals to 1 because the indifference curve is partitioned in such a way that the consumer is getting only one more unit of X each time. Thus MRS_{XY} can be directly read from the change in Y. If the consumer currently has 5 units of X, his/her MRS is 5 (The consumer is willing to give 5 units of Y in order to get one more unit of X). If the consumer currently has 7 units of X, his or her MRS is 1.



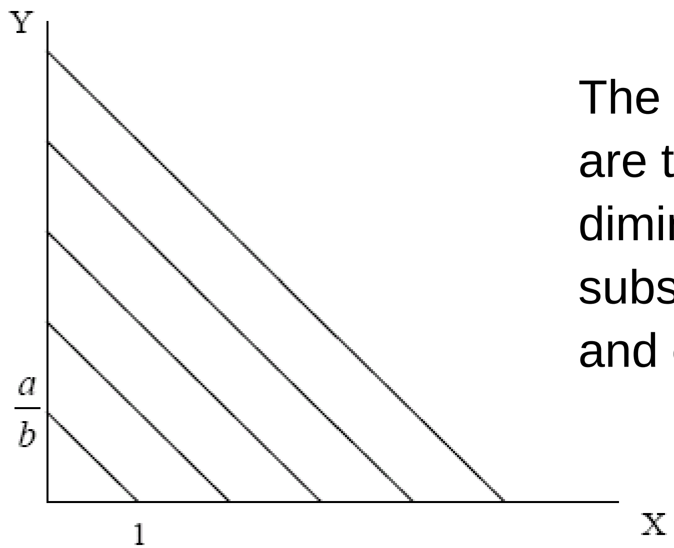
Examples of Utility Function and Indifference Curves

Perfect Substitution

The utility function for someone towards two perfect substitutes (*X and Y) is given by:*

$$u(X,Y) = aX + bY$$

That is, for this individual, only the total amount of these two goods matters. Graphically, the indifference curves look like



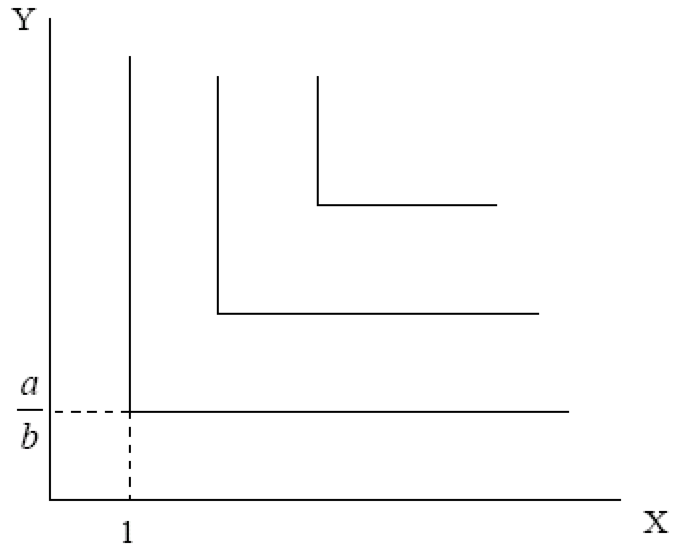
The slope of all these indifference curves are the same, -a/b . We do not have diminishing MRS in the case of perfect substitution because MRS is also constant and equals to a/b

Examples of Utility Function and Indifference Curves

Perfect complement

The utility function for someone towards two perfect complements is given by: $u(X,Y) = \min\{ax, by\}$

That is, for this individual, only the smallest amount of these two goods matters. A widely cited example of perfect complement is right foot shoes and left foot shoes. A hundred right foot shoes and one left foot shoe only makes one pair of shoes and only works for one normal person. Graphically speaking, the indifference curves look like:



Examples of Utility Function and Indifference Curves

The slope of the indifference curves, as well as MRS, can be calculated according to the following rules:

(1) At the turning point of each indifference curve, slope of MRS does not exist;

(2) Above the turning point, the value of slope and MRS are infinity;

(3) On the right of the turning point, the value of slope and MRS are zero.

Mathematically speaking, you cannot calculate the slope of the indifference curve (thus MRS) at the turning point because at that point the (utility) function is discontinued.

Examples of Utility Function and Indifference Curves

Cobb-Douglas Utility Function

Charles Cobb was a mathematician at Amherst College, and Paul Douglas was an economist at the University of Chicago. In 1928, they published a paper titled “A Theory of Production” on *American Economic Review, proposed the following functional form that can be used to capture the relationship between two different inputs of production:*

$$Output = X^a Y^b$$

Normally, the exponents a and bsatisfied

$$a > 0$$
$$b > 0$$
$$a + b = 1$$

In terms of utility function, a Cobb-Douglas utility function is written as

$$u(X, Y) = X^a Y^b$$

Examples of Utility Function and Indifference Curves

Cobb-Douglas Utility Function

The indifference curves given by the Cobb-Douglas utility function satisfy all the four characteristics introduced in this lecture. Its wide use is partly because of its goodness in fitting the theory and data. The well-behaved indifference curve we've seen is from a Cobb-Douglas utility function.

Utility Maximization in Energy Modeling

ETA-MACRO

MARKAL-MACRO

TIMES-MACRO

BUEMS-MACRO

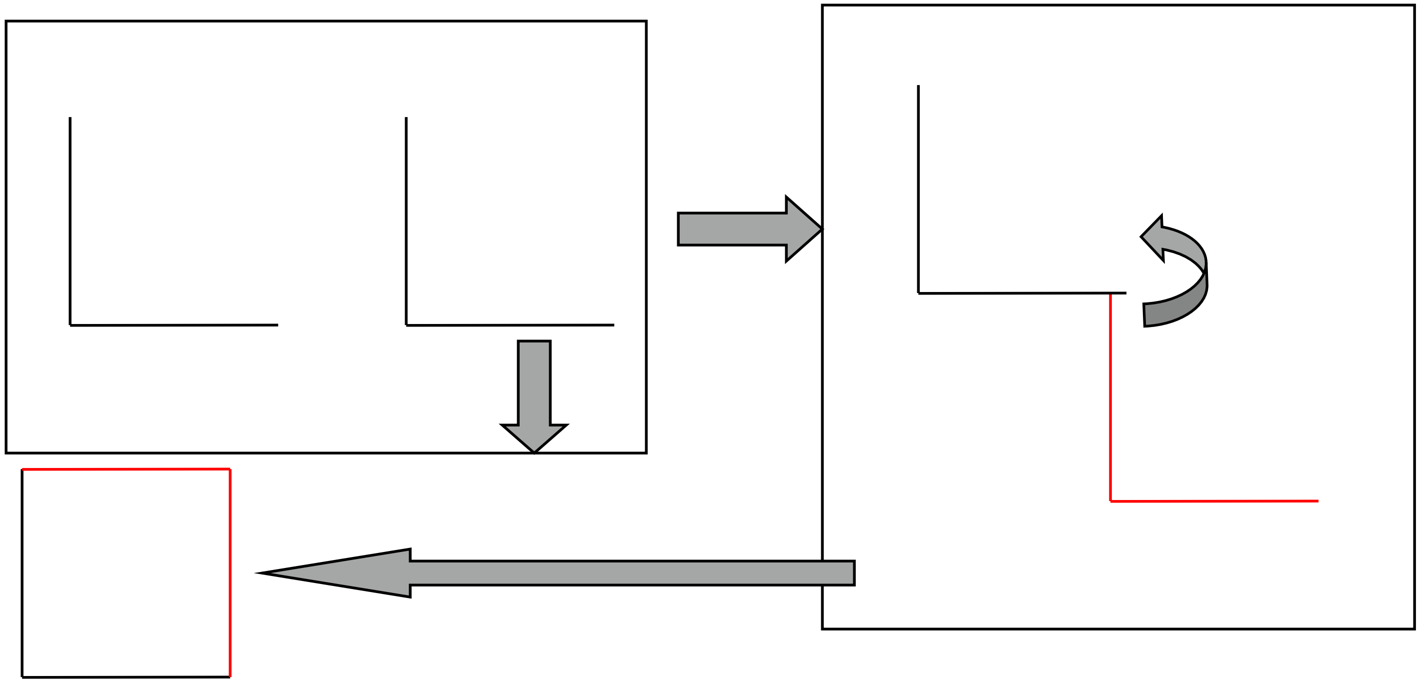
…….-MACRO

Edgeworth Box

- The prices of substitutes and complements for a good will influence the demand for it, and, the prices of goods that people sell will affect the amount of income they have and thereby influence much of other goods they will be able to buy
- How do demand and supply conditions interact in several markets ?
- Francis Edgeworth developed this method of analysis in the last portion of the 19th century.
- Provides a powerful way of graphically studying exchange and the role of markets.

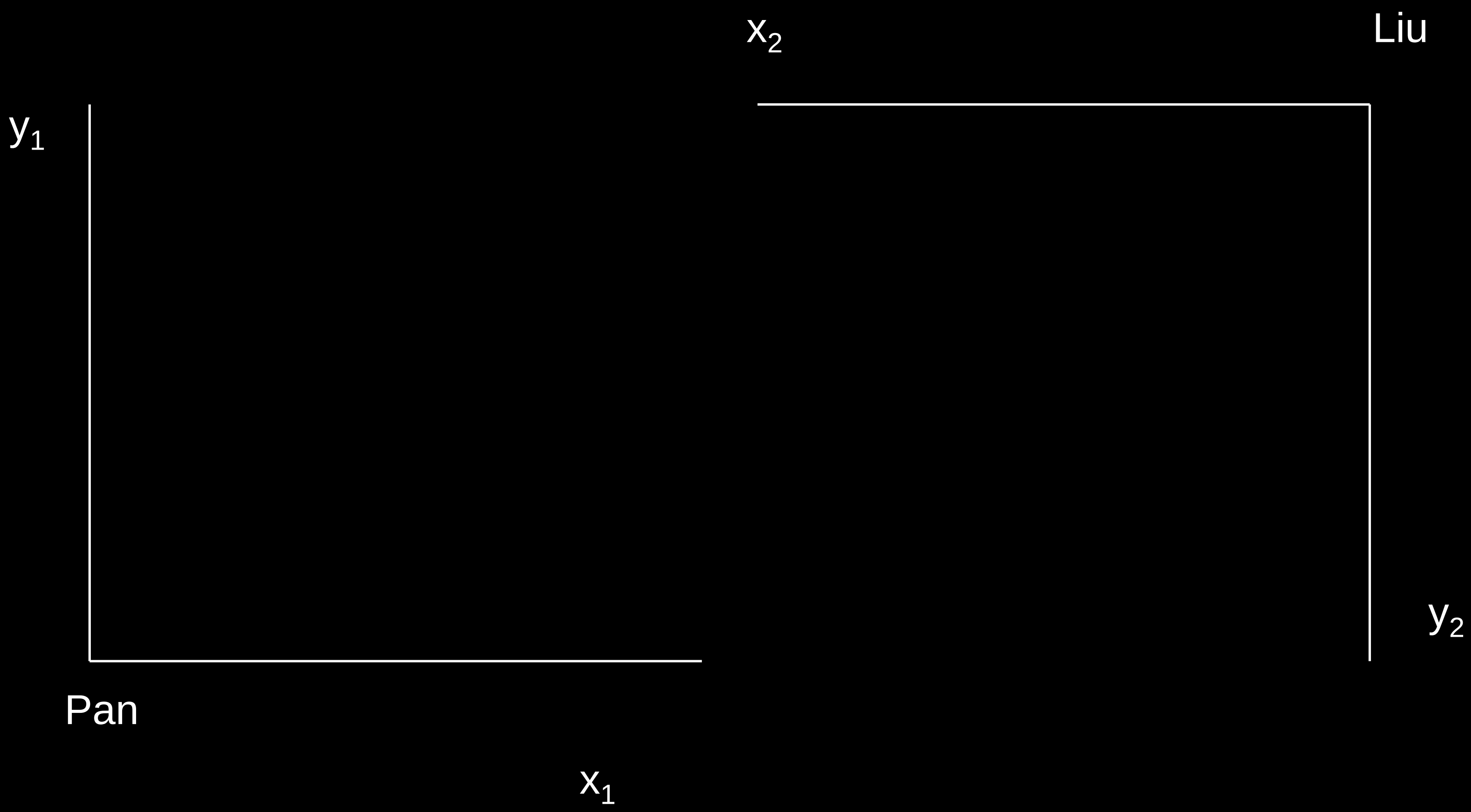
Edgeworth Box

- Simple economy: two goods, two consumers: **Liu and Pan**
- Rotate one of the graphs onto the other one until it forms a box.



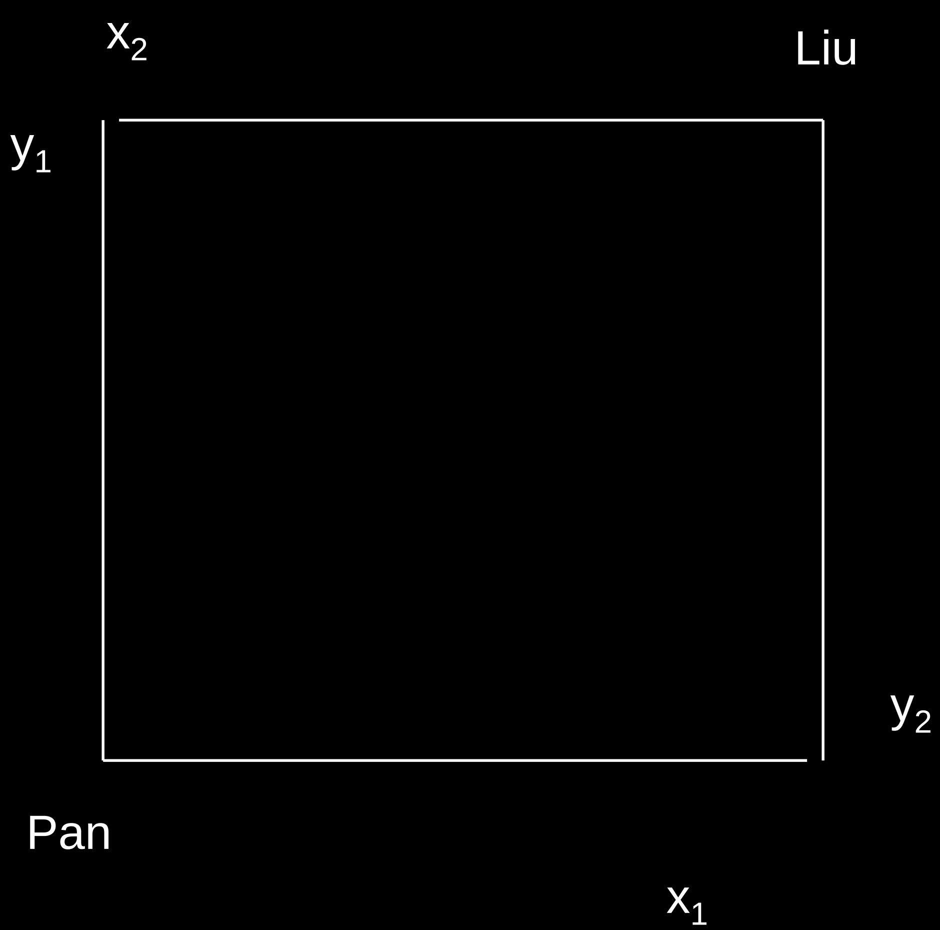
Utility Function: Theoretical Underpinnings

Here the axes for Liu have been rotated

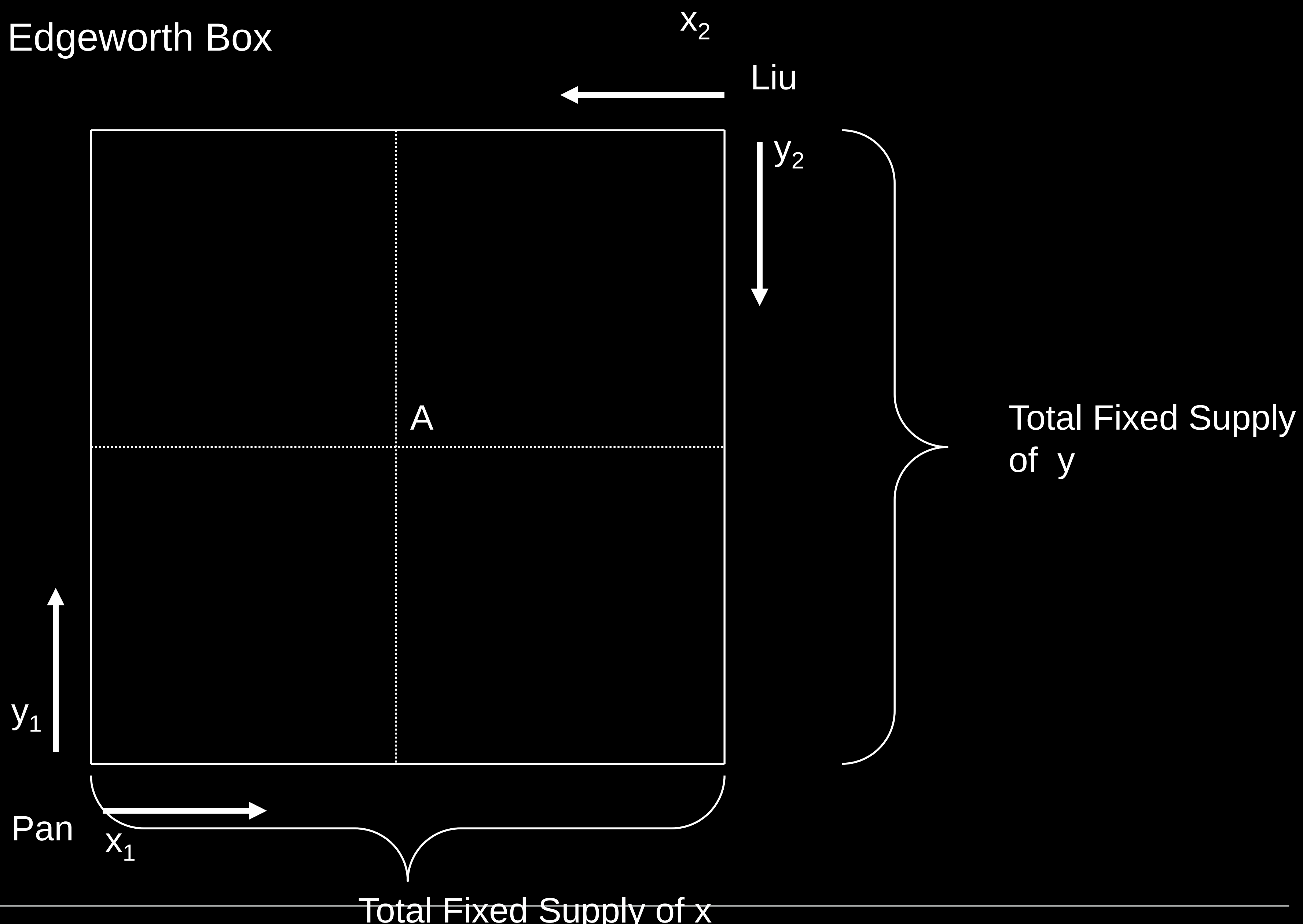


Utility Function: Theoretical Underpinnings

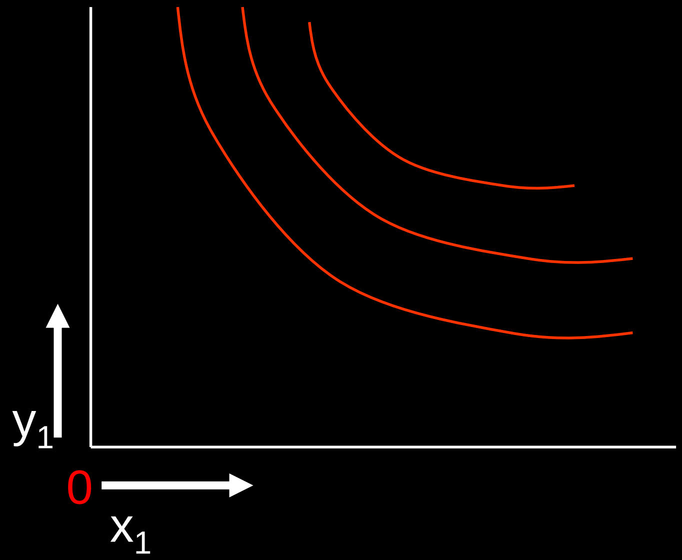
Move axes for Liu to close box



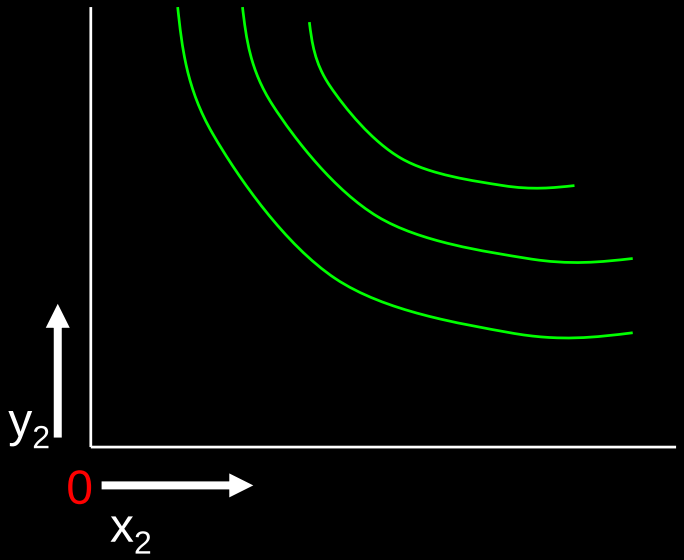
The Edgeworth Box



Consider Liu and Pan's Indifference Curves for two products

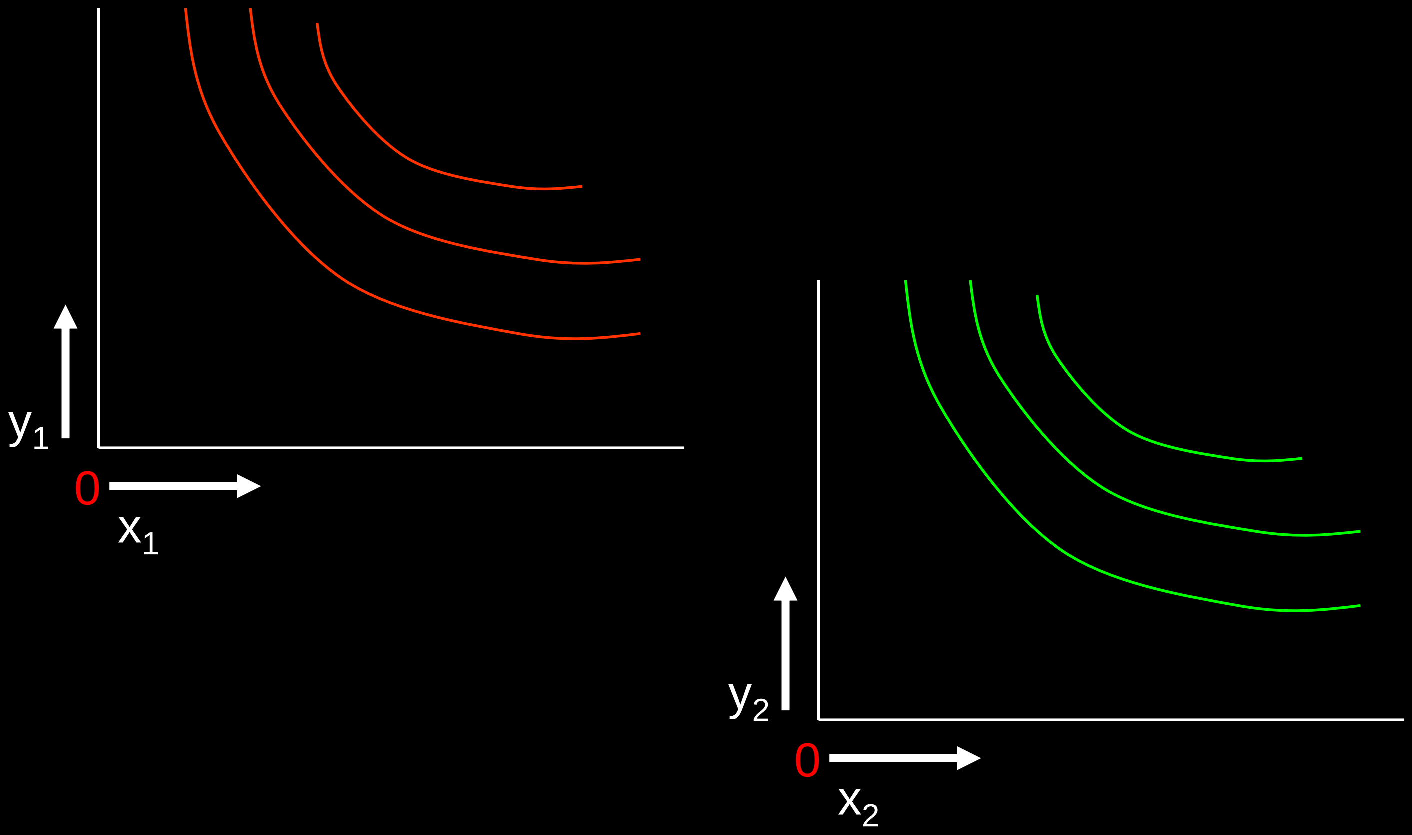


Pan

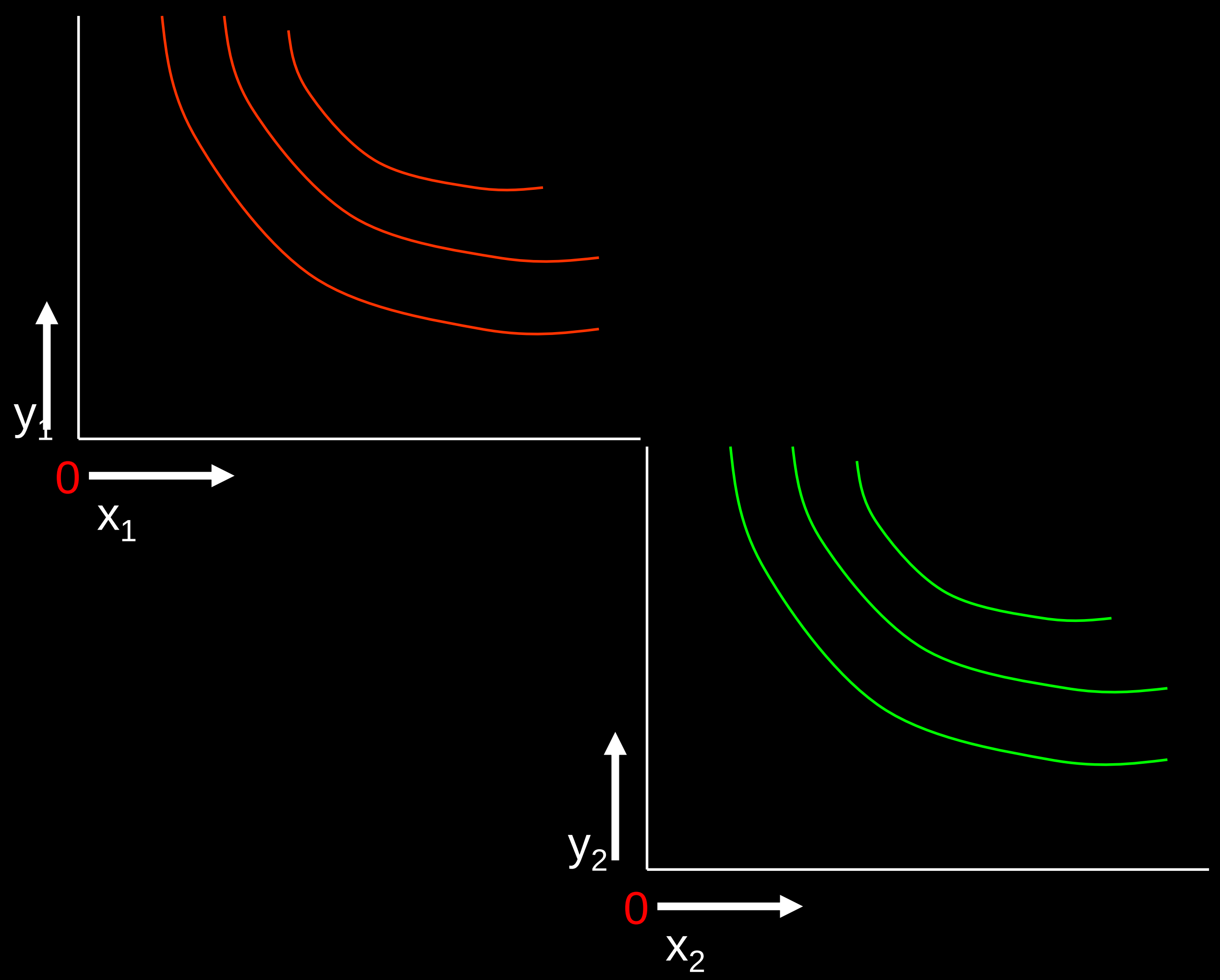


Liu

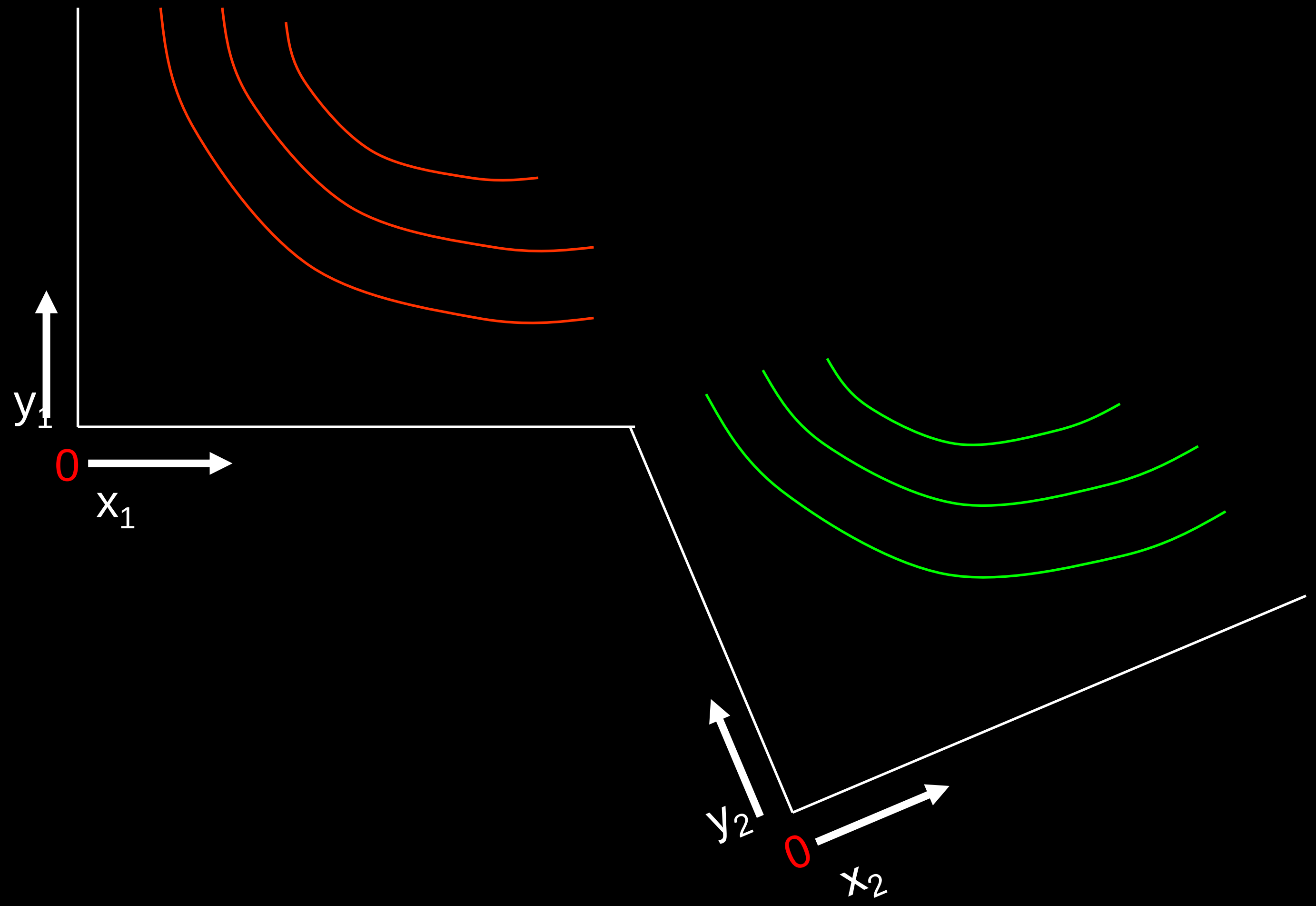
The Edgeworth Box



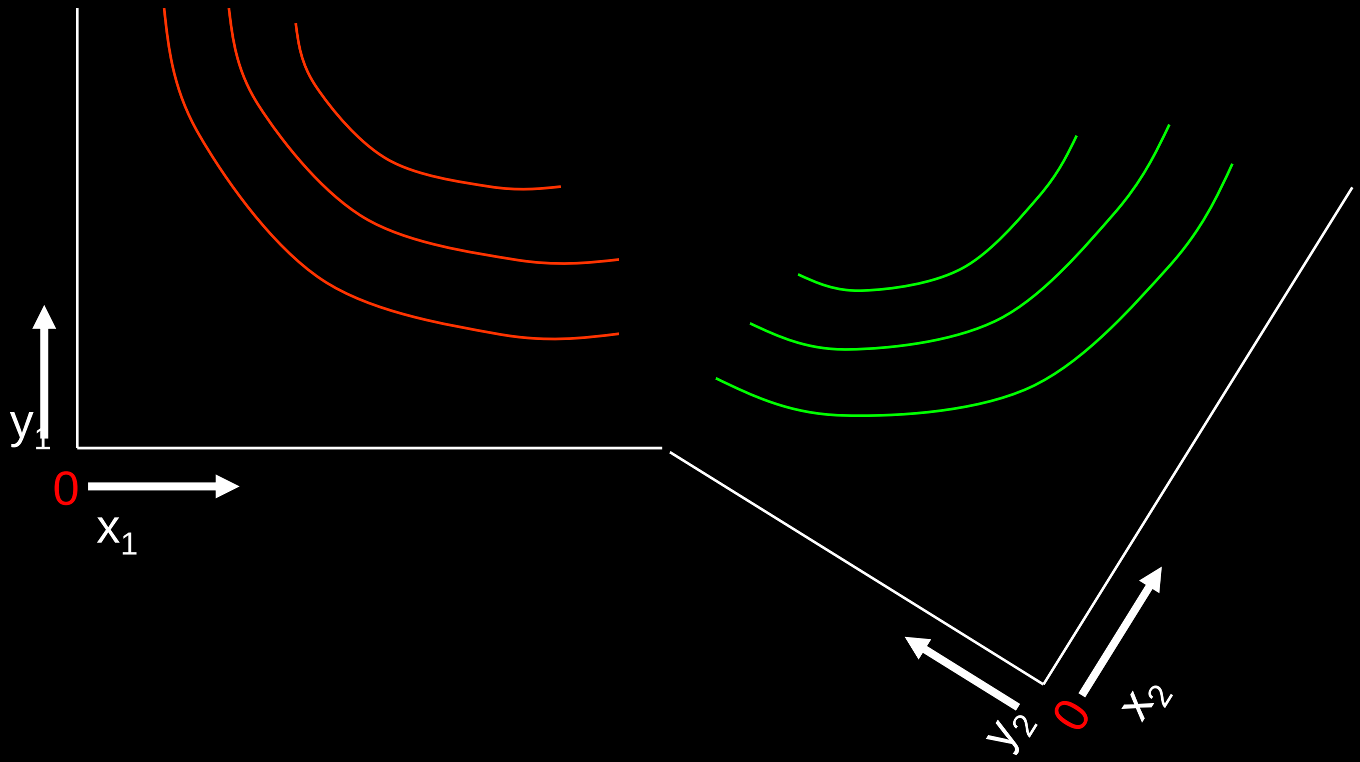
The Edgeworth Box



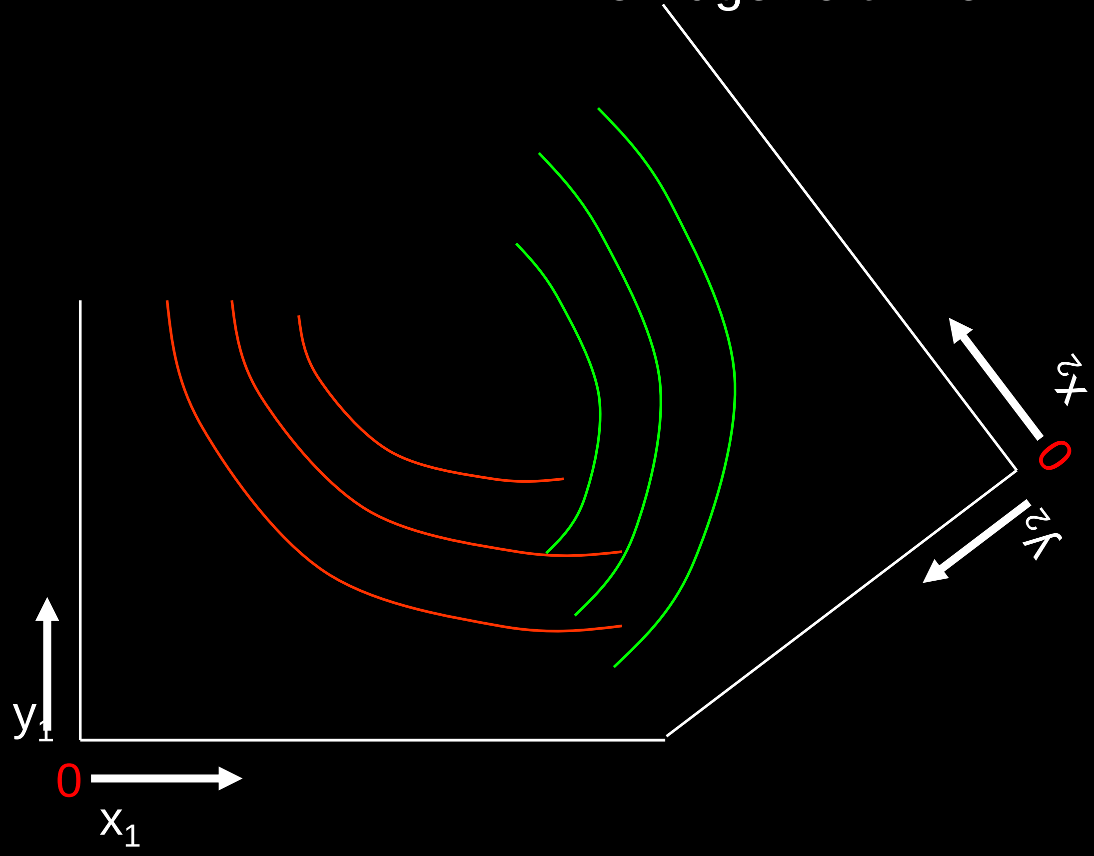
The Edgeworth Box



The Edgeworth Box



The Edgeworth Box



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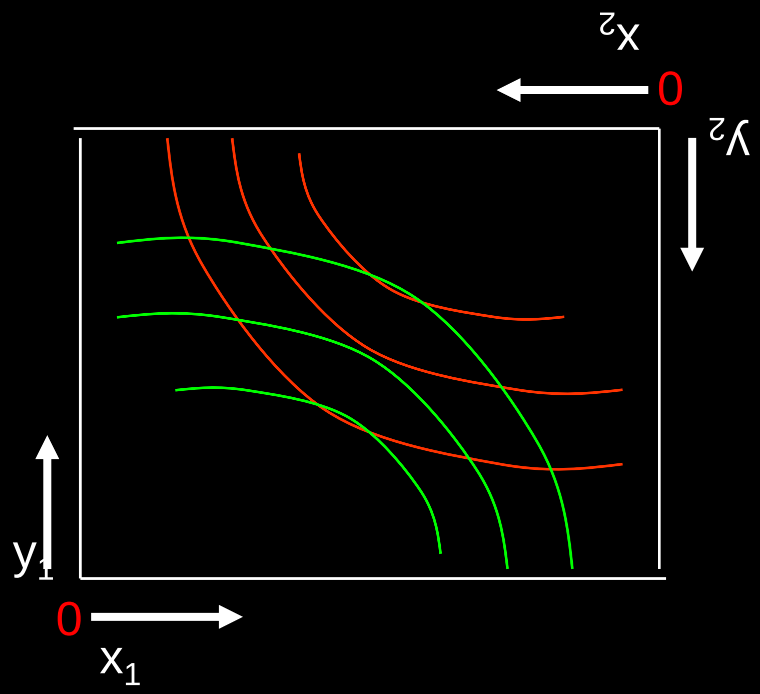
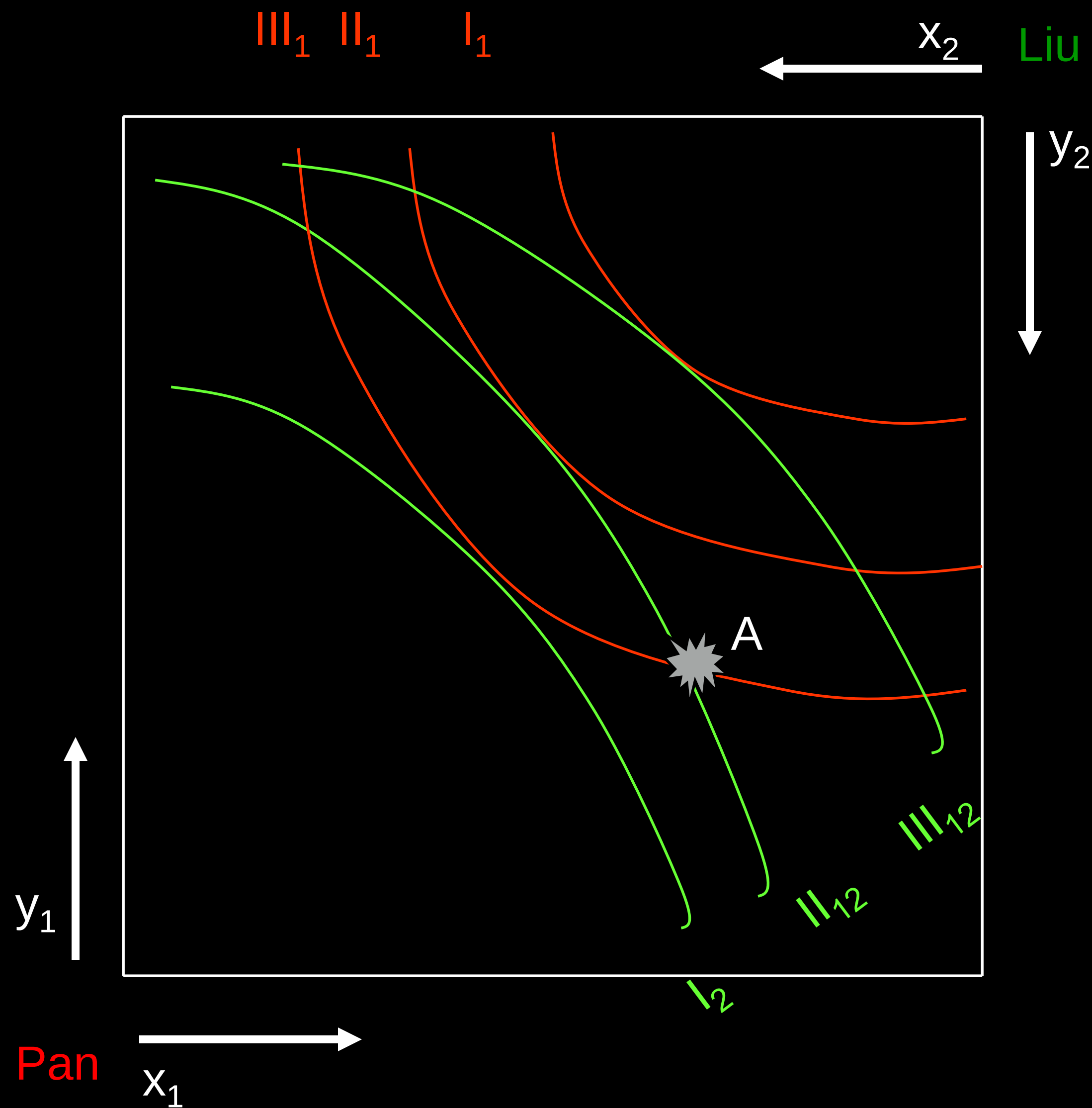


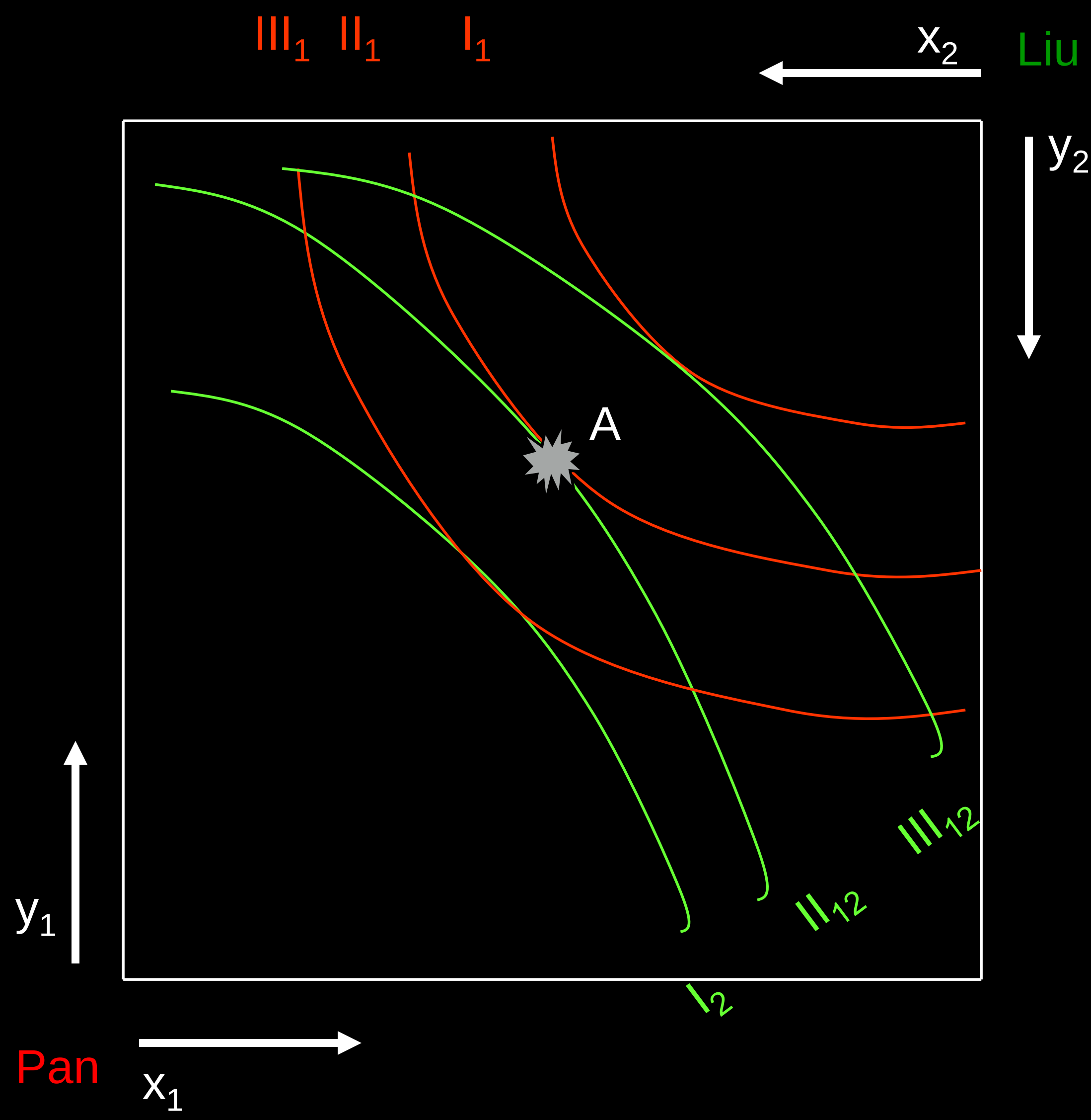
The Edgeworth Box



Intial endowment A

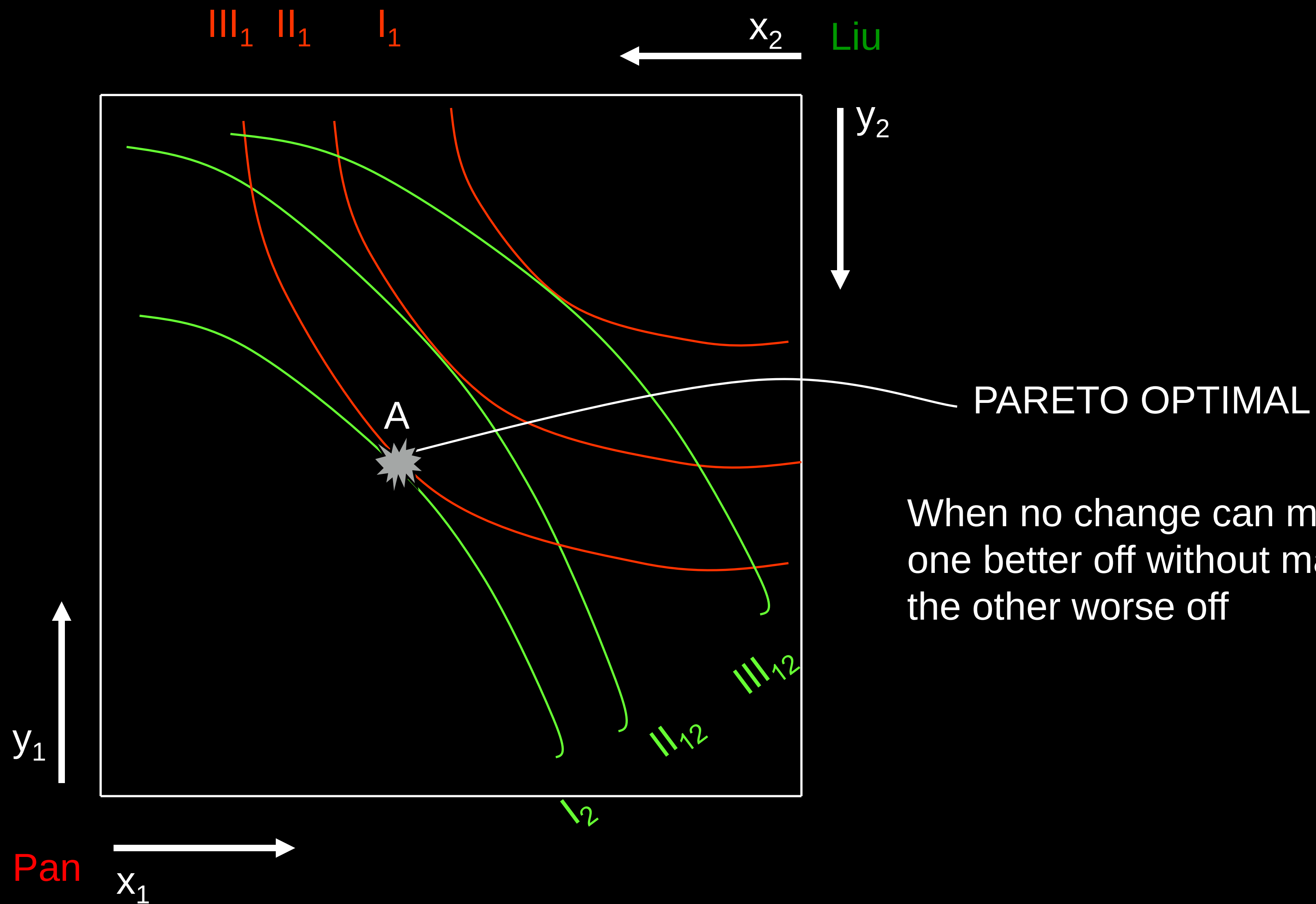
Trading area ?

The Edgeworth Box



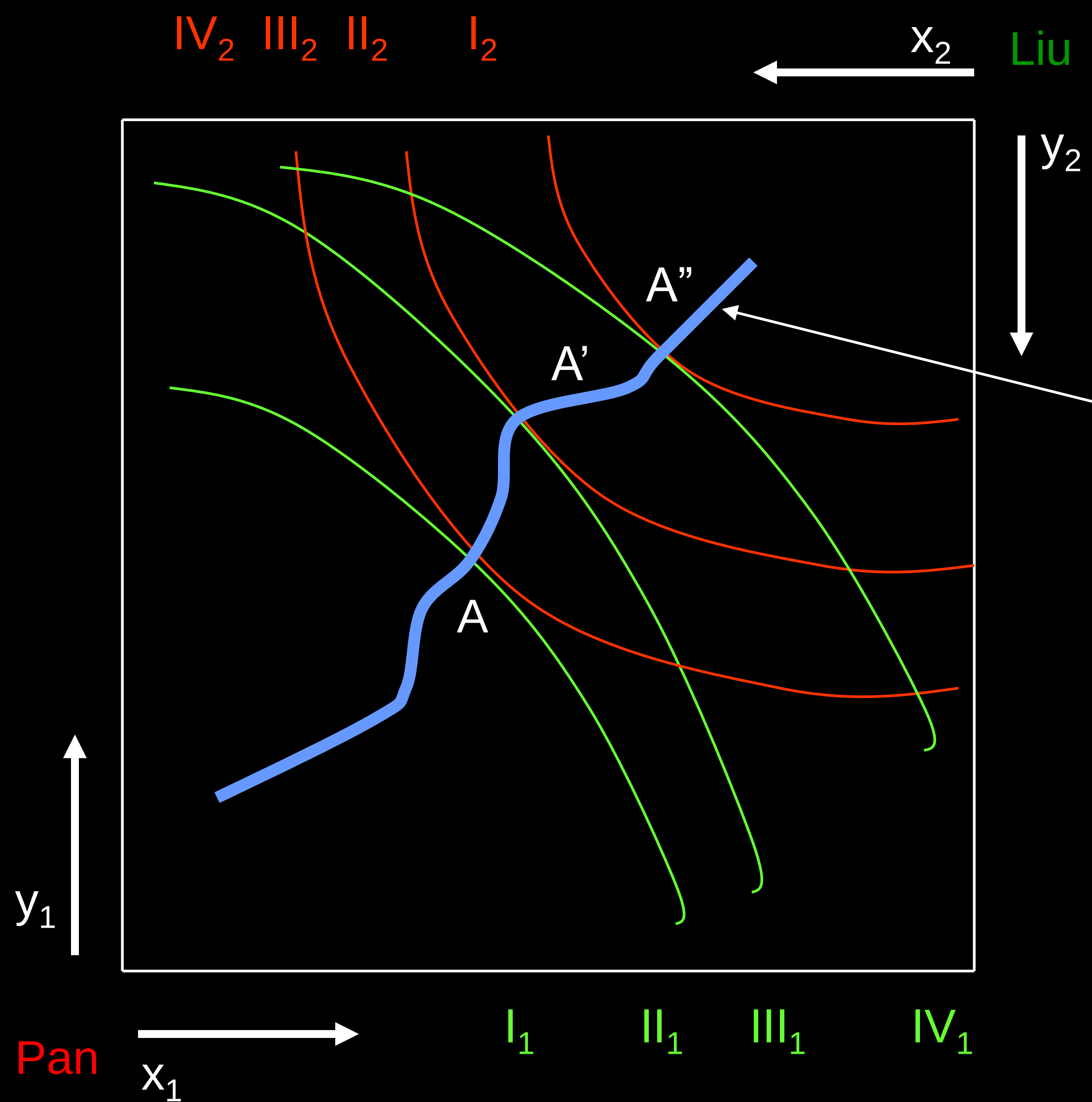
What about A here?

The Edgeworth Box



When no change can make one better off without making the other worse off

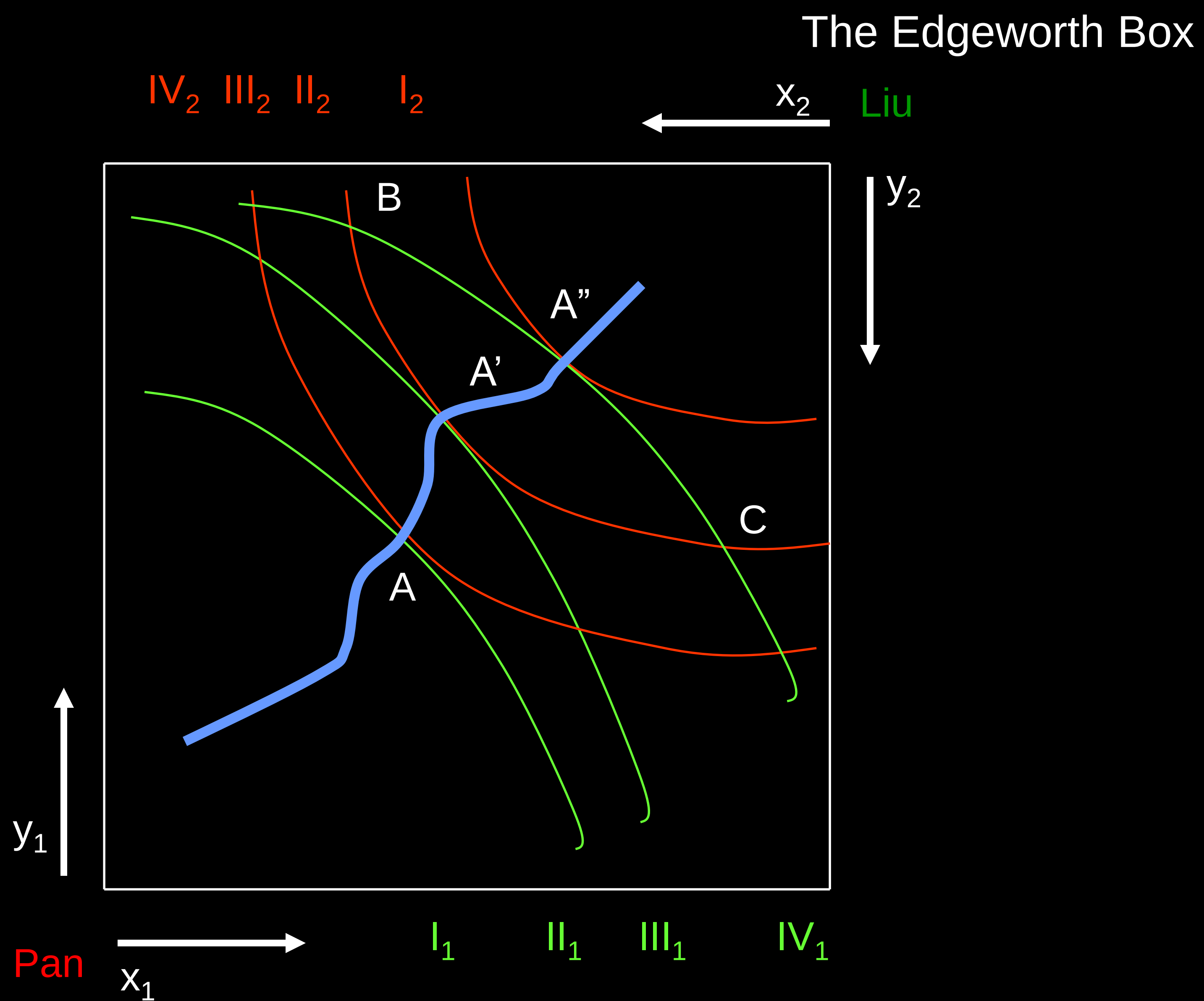
The Edgeworth Box



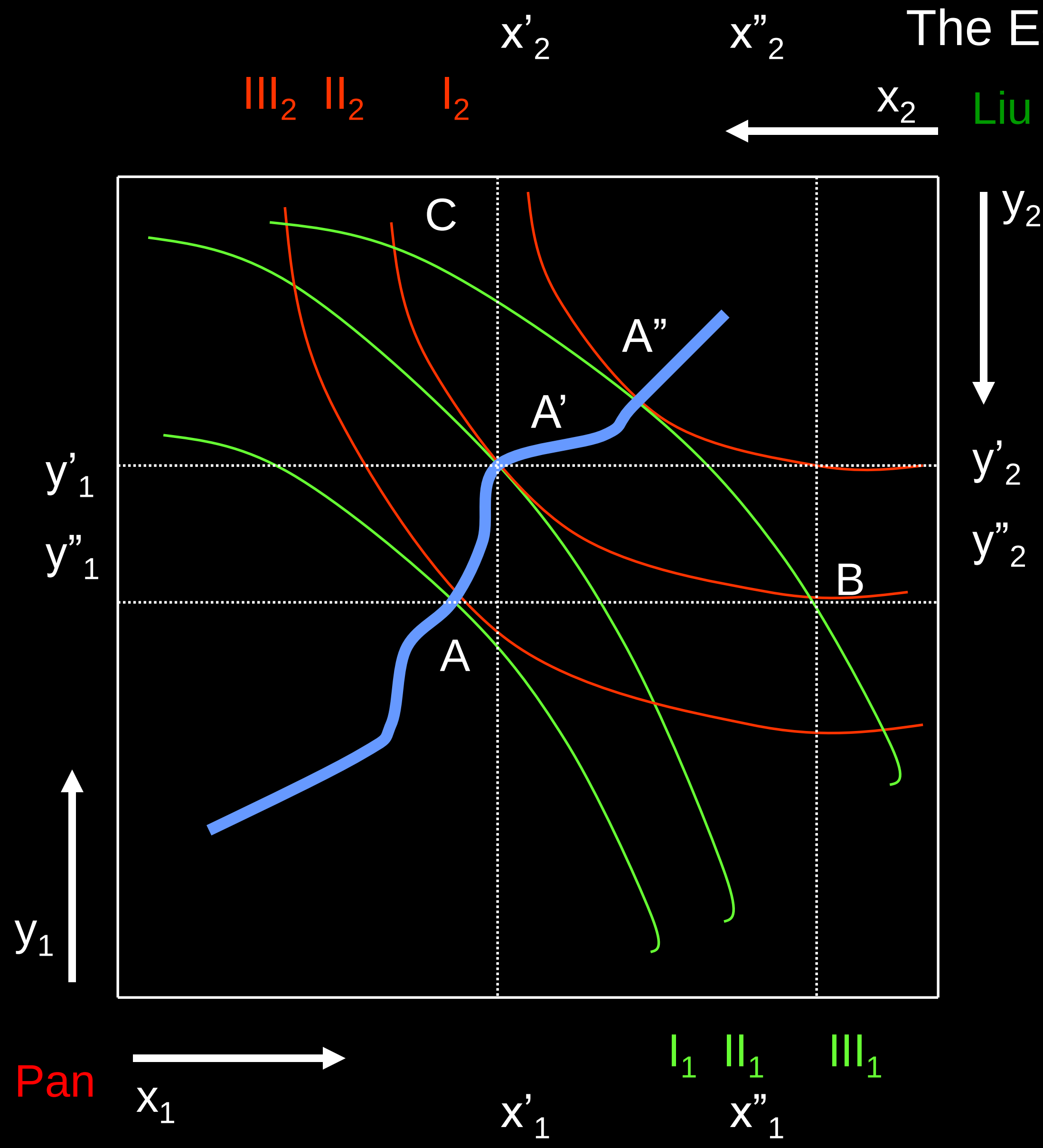
Contract line

Is the locus of Pareto optimal points

Utility Function: Theoretical Underpinnings



The Edgeworth Box

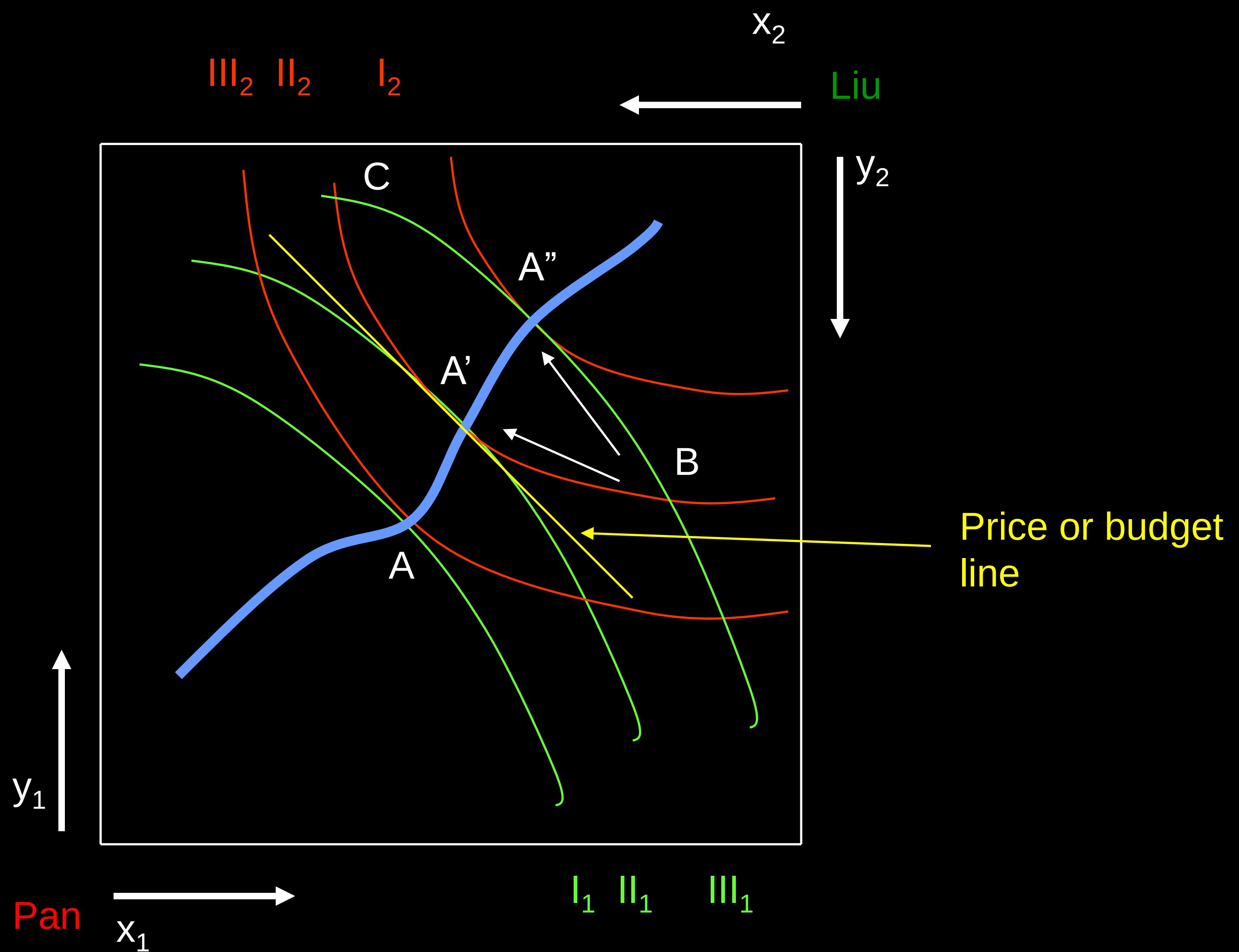


Pareto improving--from B or C to A or A"

Edgeworth Box

- Any point in the Edgeworth box indicates a particular distribution of the two goods among Pan and Liu
- Each individual has an indifference curve going through that point.
- If the distribution is Pareto optimal, those two indifference curves are tangent at that point.
- At that tangency of the two indifference curves, the slope of the tangency line--the straight line drawn through the point of tangency--represents the relative prices for the two goods. Hence, there are relative prices that will be consistent with the Pareto optimum.

Utility Function: Theoretical Underpinnings



Edgeworth Box

Tangent line is really a budget line for both individuals

- If one extends the tangent line to each axis, we now have a budget line.
- For example, the budget line for Liu is

$$I_{Liu} = P_x x_{Liu} + P_y y_{Liu}$$

where I is the income Liu could get from selling the X and Y she holds at the Pareto optimum point.

Utility Function: Theoretical Underpinnings